Classroom Voting Questions: Differential Equations and Linear Algebra

Chapter 1: First-Order Differential Equations

What is a Differential Equation?

1. Which of the following is not a differential equation?

   (a) $y' = 3y$
   (b) $2x^2y + y^2 = 6$
   (c) $tx\frac{dx}{dt} = 2$
   (d) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 7y + 8x = 0$
   (e) All are differential equations.

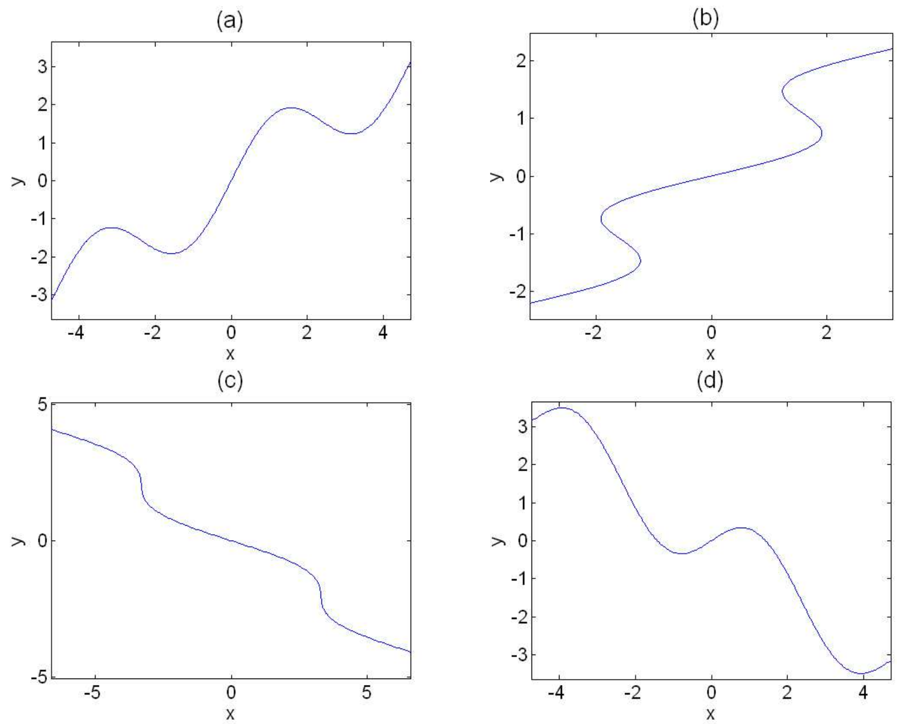
2. Which of the following is not a differential equation?

   (a) $6\frac{dy}{dx} + 3xy$
   (b) $8 = \frac{y'}{y}$
   (c) $2\frac{d^2f}{dt^2} + 7\frac{df}{dt} = f$
   (d) $h(x) + 2h'(x) = g(x)$
   (e) All are differential equations.

3. Which of the following couldn't be the solution of a differential equation?

   (a) $z(t) = 6$
   (b) $y = 3x^2 + 7$
   (c) $x = 0$
   (d) $y = 3x + y'$
   (e) All could be solutions of a differential equation.

4. Which of the following could not be a solution of a differential equation?



5. Which of the following could not be a solution of a differential equation?

   (a) $f = 2y + 7$
   (b) $q(d) = 2d^2 - 6e^d$
   (c) $6y^2 + 2yx = \sqrt{x}$
   (d) $y = 4 \sin 8\pi z$
   (e) All could be a solution of a differential equation.

6. True or False? A differential equation is a type of function.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

7. Suppose $\dfrac{dx}{dt} = 0.5x$ and $x(0) = 8$. Then the value of $x(2)$ is approximately

   (a) 4
   (b) 8

(c) 9
(d) 12
(e) 16

8. Which of the following is a solution to the differential equation $\dfrac{dy}{dt} = 72 - y$?

   (a) $y(t) = 72t - \frac{1}{2}t^2$
   (b) $y(t) = 72 + e^{-t}$
   (c) $y(t) = e^{-72t}$
   (d) $y(t) = e^{-t}$

9. The amount of a chemical in a lake is decreasing at a rate of 30% per year. If $p(t)$ is the total amount of the chemical in the lake as a function of time t (in years), which differential equation models this situation?

   (a) $p'(t) = -30$
   (b) $p'(t) = -0.30$
   (c) $p'(t) = p - 30$
   (d) $p'(t) = -0.3p$
   (e) $p'(t) = 0.7p$

10. The evolution of the temperature of a hot cup of coffee cooling off in a room is described by $\frac{dT}{dt} = -0.01T + 0.6$, where T is in °F and t is in hours. What are the units of the numbers -0.01 and 0.6?

    (a) -0.01 °F, and 0.6 °F
    (b) -0.01 per hour, and 0.6 °F per hour
    (c) -0.01 °F per hour, and 0.6 °F
    (d) neither number has units

11. We want to test the function $z(x) = 4\sin 3x$ to see if it solves $z'' + 2z' + 4z = 0$, by substituting the function into the differential equation. What is the resulting equation before simplification?

    (a) $-36\sin 3x + 24\cos 3x + 16\sin 3x = 0$
    (b) $4\sin 3x + 8\sin 3x + 16\sin 3x = 0$
    (c) $-36\sin 3x + 12\cos 3x + 4\sin 3x = 0.$

(d) $4\sin 3x + 8\cos 3x + 4\sin 3x = 0$

(e) none of the above

12. If we test the function $f(x) = ae^{bx}$ to see if it could solve $\frac{df}{dx} = cf^2$, which equation is the result?

(a) $\frac{df}{dx} = ca^2e^{2bx}$

(b) $abe^{bx} = cf^2$

(c) $ae^{bx} = ca^2e^{(bx)^2}$

(d) $abe^{bx} = ca^2e^{2bx}$

(e) $abe^{bx} = cae^{bx}$

(f) None of the above

13. We want to test the function $f(x) = 3e^{2x} + 6x$ to see if it solves the differential equation $\frac{df}{dx} = 2f + 3x$, so we insert the function and its derivative, getting $6e^{2x} + 6 = 2(3e^{2x} + 6x) + 3x$. This means that:

(a) This function is a solution.

(b) This function is a solution if $x = 2/5$.

(c) This function is not a solution.

(d) Not enough information is given.

14. A bookstore is constantly discarding a certain percentage of its unsold inventory and also receiving new books from its supplier so that the rate of change of the number of books in inventory is $B'(t) = -0.02B + 400 + 0.05t$, where B is the number of books and t is in months. If the store begins with 10,000 books in inventory, at what rate is it receiving books from its supplier at $t = 0$?

(a) 200 books per month

(b) 400 books per month

(c) -200 books per month

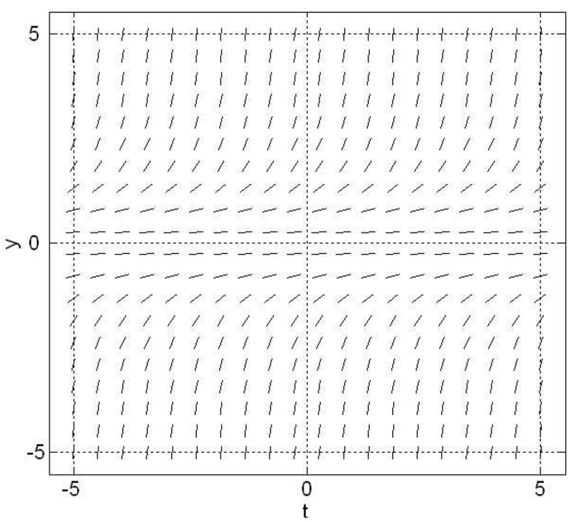
(d) 900 books per month

Slope Fields and Euler's Method

15. What does the differential equation $\frac{dy}{dx} = 2y$ tell us about the slope of the solution curves at at any point?

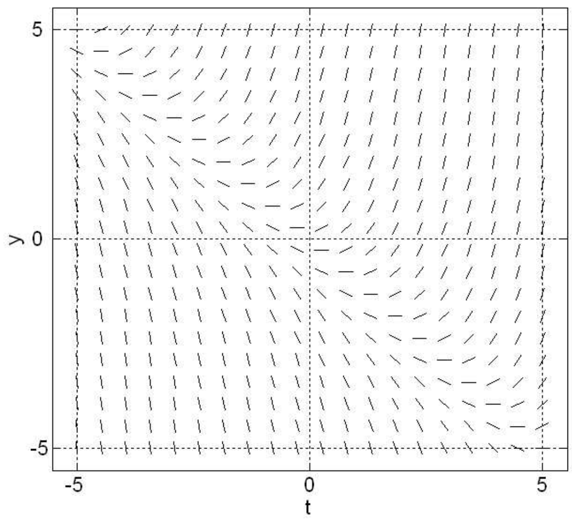
    (a) The slope is always 2.
    (b) The slope is equal to the x-coordinate.
    (c) The slope is equal to the y-coordinate.
    (d) The slope is equal to two times the x-coordinate.
    (e) The slope is equal to two times the y-coordinate.
    (f) None of the above.

16. The slopefield below indicates that the differential equation has which form?



    (a) $\frac{dy}{dt} = f(y)$
    (b) $\frac{dy}{dt} = f(t)$
    (c) $\frac{dy}{dt} = f(y, t)$

17. The slopefield below indicates that the differential equation has which form?

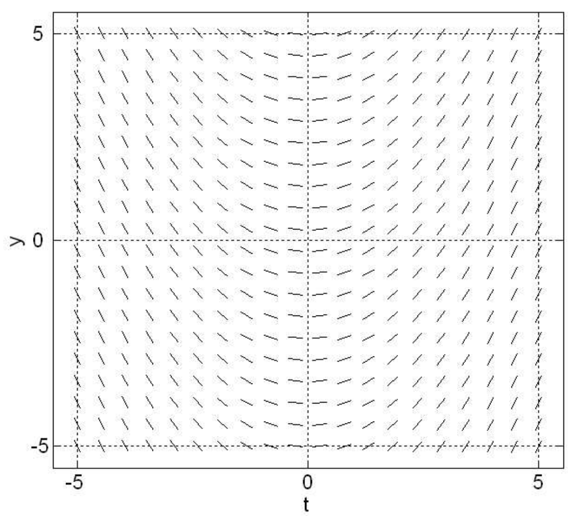


(a) $\frac{dy}{dt} = f(y)$

(b) $\frac{dy}{dt} = f(t)$

(c) $\frac{dy}{dt} = f(y, t)$

18. The slopefield below indicates that the differential equation has which form?

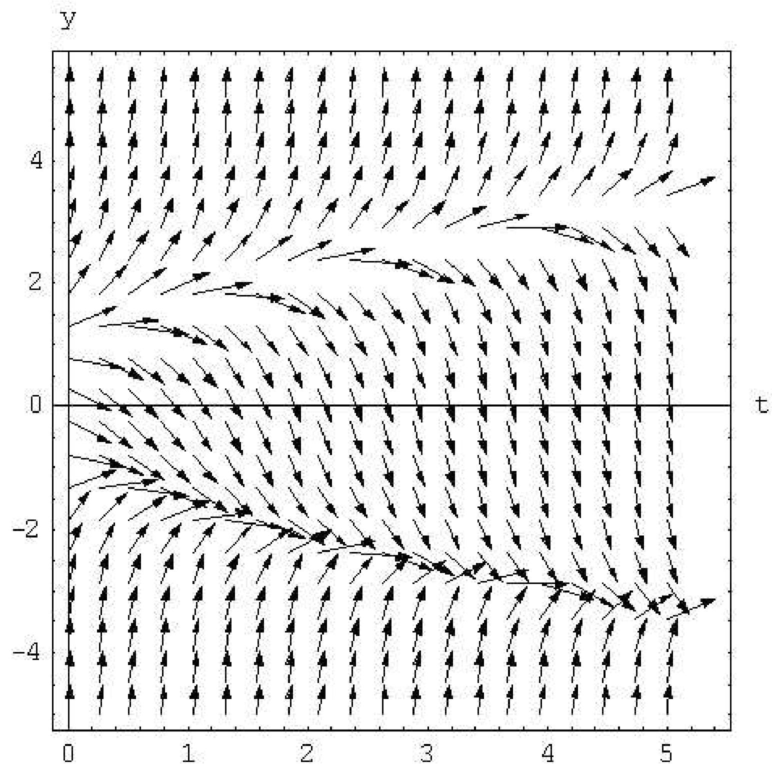


(a) $\frac{dy}{dt} = f(y)$

(b) $\frac{dy}{dt} = f(t)$

(c) $\frac{dy}{dt} = f(y, t)$

19. The arrows in the slope field below have slopes that match the derivative y' for a range of values of the function y and the independent variable t. Suppose that $y(0) = 0$. What would you predict for $y(5)$?



(a) $y(5) \approx -3$

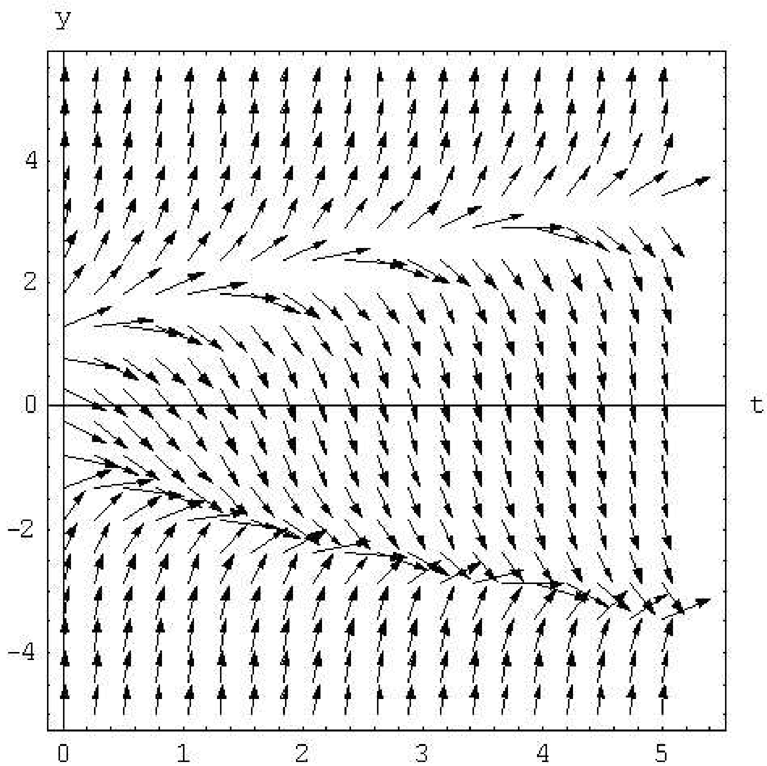
(b) $y(5) \approx +3$

(c) $y(5) \approx 0$

(d) $y(5) < -5$

(e) None of the above

20. The arrows in the slope field below give the derivative y' for a range of values of the function y and the independent variable t. Suppose that $y(0) = -4$. What would you predict for $y(5)$?



(a) $y(5) \approx -3$

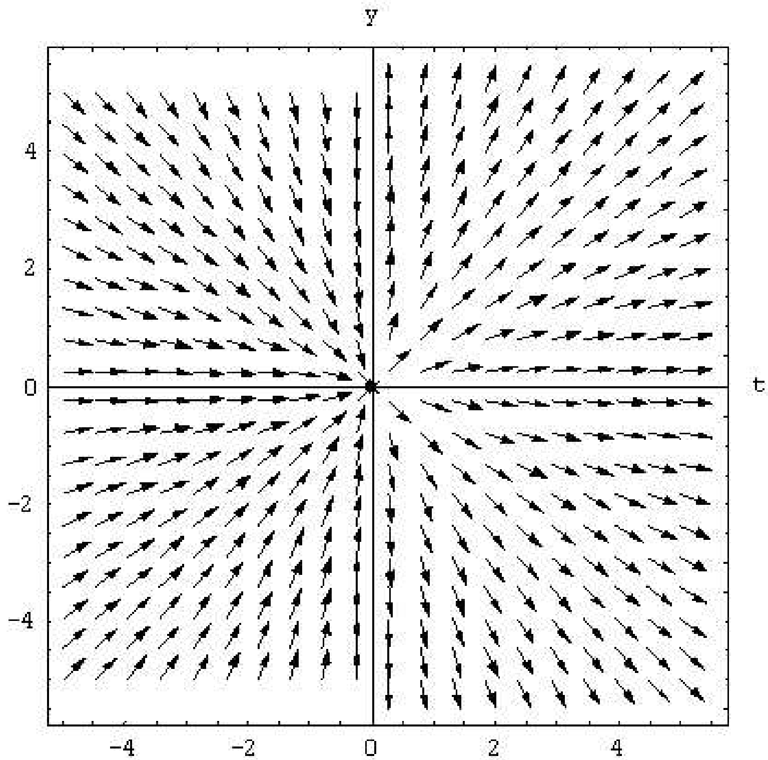
(b) $y(5) \approx +3$

(c) $y(5) \approx 0$

(d) $y(5) < -5$

(e) None of the above

21. The slope field below represents which of the following differential equations?



(a) $y' = yt$

(b) $y' = \frac{y}{t}$

(c) $y' = -yt$

(d) $y' = -\frac{y}{t}$

22. Consider the differential equation $y' = ay + b$ with parameters a and b. To approximate this function using Euler's method, what difference equation would we use?

(a) $y_{n+1} = ay_n + b$

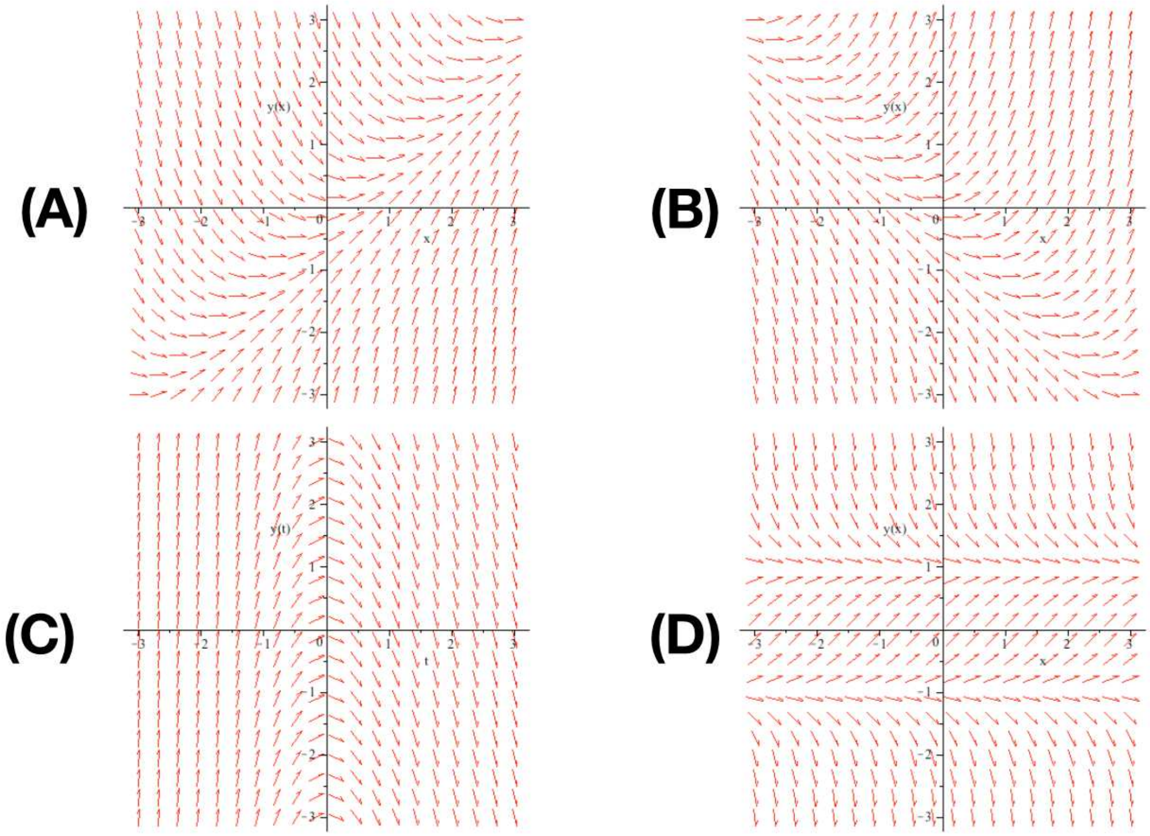
(b) $y_{n+1} = y_n + ay_n\Delta t + b\Delta t$

(c) $y_{n+1} = ay_n\Delta t + b\Delta t$

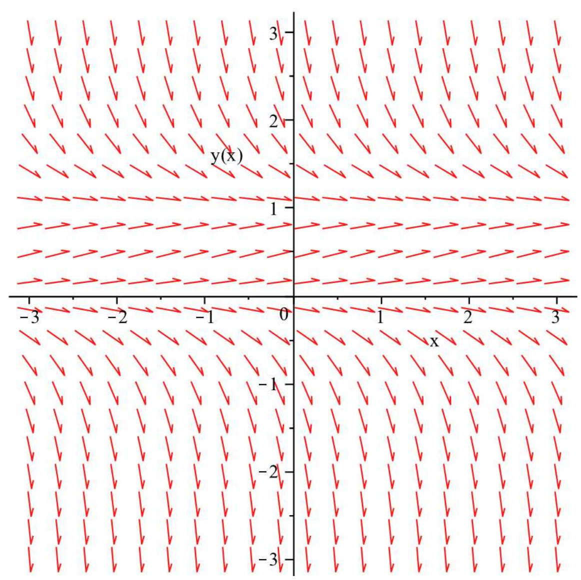
(d) $y_{n+1} = y_n\Delta t + ay_n\Delta t + b\Delta t$

(e) None of the above

23. Which of the following is the slope field for $dy/dx = x + y$?



24. Below is the slope field for $dy/dx = y(1-y)$:



As $x \to \infty$, the solution to this differential equation that satisfies the initial condition $y(0) = 2$ will

(a) Increase asymptotically to $y = 1$

(b) Decrease asymptotically to $y = 1$

(c) Increase without bound

(d) Decrease without bound

(e) Start and remain horizontal

25. Using Euler's method, we set up the difference equation $y_{n+1} = y_n + c\Delta t$ to approximate a differential equation. What is the differential equation?

(a) $y' = cy$

(b) $y' = c$

(c) $y' = y + c$

(d) $y' = y + c\Delta t$

(e) None of the above

26. We know that $f(2) = -3$ and we use Euler's method to estimate that $f(2.5) \approx -3.6$, when in reality $f(2.5) = -3.3$. This means that between $x = 2$ and $x = 2.5$,

(a) $f(x) > 0$.

(b) $f'(x) > 0$.

(c) $f''(x) > 0$.

(d) $f'''(x) > 0$.

(e) None of the above

27. We have used Euler's method and $\Delta t = 0.5$ to approximate the solution to a differential equation with the difference equation $y_{n+1} = y_n + 0.2$. We know that the function $y = 7$ when $t = 2$. What is our approximate value of y when $t = 3$?

(a) $y(3) \approx 7.2$

(b) $y(3) \approx 7.4$

(c) $y(3) \approx 7.6$

(d) $y(3) \approx 7.8$

(e) None of the above

28. We have used Euler's method to approximate the solution to a differential equation with the difference equation $z_{n+1} = 1.2z_n$. We know that the function $z(0) = 3$. What is the approximate value of $z(2)$?

(a) $z(2) \approx 3.6$
(b) $z(2) \approx 4.32$
(c) $z(2) \approx 5.184$
(d) Not enough information is given.

29. We have used Euler's method and $\Delta t = 0.5$ to approximate the solution to a differential equation with the difference equation $y_{n+1} = y_n + t + 0.2$. We know that the function $y = 7$ when $t = 2$. What is our approximate value of y when $t = 3$?

(a) $y(3) \approx 7.4$
(b) $y(3) \approx 11.4$
(c) $y(3) \approx 11.9$
(d) $y(3) \approx 12.9$
(e) None of the above

30. We have a differential equation for $\frac{dx}{dt}$, we know that $x(0) = 5$, and we want to know $x(10)$. Using Euler's method and $\Delta t = 1$ we get the result that $x(10) \approx 25.2$. Next, we use Euler's method again with $\Delta t = 0.5$ and find that $x(10) \approx 14.7$. Finally we use Euler's method and $\Delta t = 0.25$, finding that $x(10) \approx 65.7$. What does this mean?

(a) These may all be correct. We need to be told which stepsize to use, otherwise we have no way to know which is the right approximation in this context.
(b) Fewer steps means fewer opportunities for error, so $x(10) \approx 25.2$.
(c) Smaller stepsize means smaller errors, so $x(10) \approx 65.7$.
(d) We have no way of knowing whether any of these estimates is anywhere close to the true value of $x(10)$.
(e) Results like this are impossible: We must have made an error in our calculations.

31. We have a differential equation for $f'(x)$, we know that $f(12) = 0.833$, and we want to know $f(15)$. Using Euler's method and $\Delta t = 0.1$ we get the result that $f(15) \approx 0.275$. Next, we use $\Delta t = 0.2$ and find that $f(15) \approx 0.468$. When we use $\Delta t = 0.3$, we get $f(15) \approx 0.464$. Finally, we use $\Delta t = 0.4$ and we get $f(15) \approx 0.462$. What does this mean?

(a) These results appear to be converging to $f(15) \approx 0.46$.
(b) Our best estimate is $f(15) \approx 0.275$.
(c) This data does not allow us to make a good estimate of $f(15)$.

Equilibria and Stability

32. The differential equation $\frac{dy}{dt} = (t-3)(y-2)$ has equilibrium values of

    (a) $y = 2$ only
    (b) $t = 3$ only
    (c) $y = 2$ and $t = 3$
    (d) No equilibrium values

33. Suppose that 3 is an equilibrium value of a differential equation. This means that

    (a) the values will approach 3.
    (b) if the initial value is below 3, the values will decrease.
    (c) if the initial value is 3, then all of the values will be 3.
    (d) all of the above.

34. We know that a given differential equation is in the form $y' = f(y)$, where f is a continuous function of y. Suppose that $f(5) = 2$ and $f(-1) = -6$.

    (a) y must have an equilibrium value between $y = 5$ and $y = -1$.
    (b) y must have an equilibrium value between $y = 2$ and $y = -6$.
    (c) This does not necessarily indicate that any equilibrium value exists.

35. We know that a given differential equation is in the form $y' = f(y)$, where f is a continuous function of y. Suppose that $f(10) = 0$, $f(9) = 3$, and $f(11) = -3$.

    (a) This means that $y = 10$ is a stable equilibrium.
    (b) $y = 10$ is an equilibrium, but it might not be stable.
    (c) This does not tell us for certain that $y = 10$ is an equilibrium.

36. We know that a given differential equation is in the form $y' = f(y)$, where f is a continuous function of y. Suppose that $f(6) = 0$, $f(14) = 0$, and $y(10) = 10$.

    (a) This means that $y(0)$ must have been between 6 and 14.
    (b) This means that $y(20) = 0$ is impossible.
    (c) This means that $y(20) = 20$ is impossible.
    (d) All of the above.

(e) None of the above.

37. We know that a given differential equation is in the form $y' = f(y)$, where f is a continuous function of y. Suppose that $f(2) = 3$ and that $y(0) = 0$. Which of the following is impossible?

    (a) $y(10) = 6$
    (b) $y(10) = -6$
    (c) $y(-10) = 6$
    (d) $y(-10) = -6$
    (e) All of these are possible

38. We know that a given differential equation is in the form $y' = f(y)$, where f is a continuous function of y. Suppose that $f(5) = -2$, $f(10) = 4$, and that $y(10) = 3$.

    (a) $y(0)$ must be below 5.
    (b) $y(20)$ must be below 5.
    (c) $y(5)$ could be above 10.
    (d) $y(15)$ must be less than 3.

39. A differential equation has a stable equilibrium value of $T = 6$. Which of the following functions is definitely not a solution?

    (a) $T(t) = 5e^{-3t} + 6$
    (b) $T(t) = -4e^{-2t} + 6$
    (c) $T(t) = 4e^{2t} + 10$
    (d) They could all be solutions

40. Consider the differential equation $\frac{df}{dx} = \sin(f)$

    (a) $f = 0$ is a stable equilibrium.
    (b) $f = 0$ is an unstable equilibrium.
    (c) $f = 0$ is not an equilibrium.

41. Consider the differential equation $\frac{df}{dx} = af + b$, where a and b are positive parameters. If we increase b, what will happen to the equilibrium value?

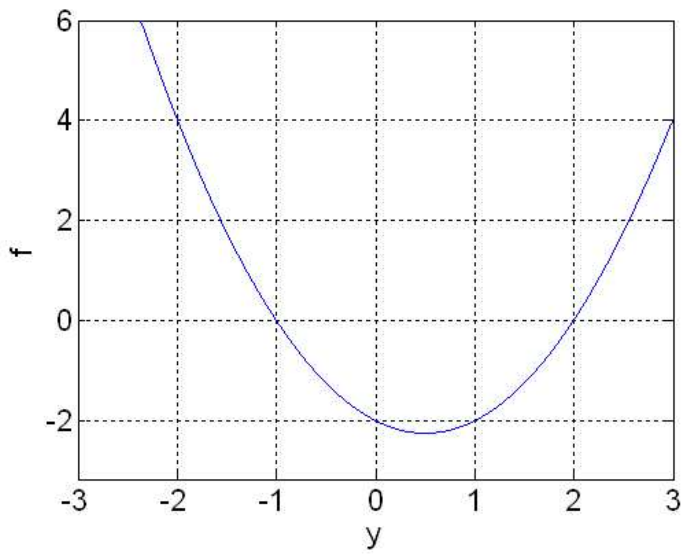
    (a) it increases

(b) is decreases

(c) it stays the same

(d) not enough information is given

42. Suppose that $\frac{dy}{dt} = f(y)$, which is plotted below. What are the equilibrium values of the system?

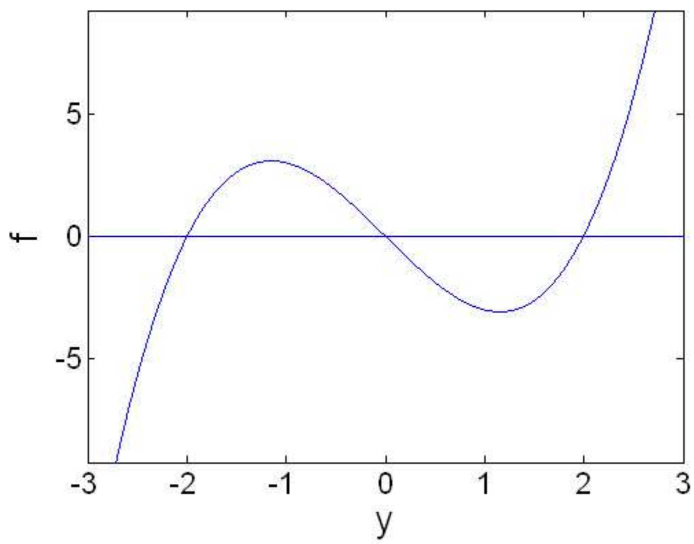


(a) $y = \frac{1}{2}$ is the only equilibrium.

(b) $y = -1$ and $y = 2$ are both equilibria.

(c) Not enough information is given.

43. Suppose that $\frac{dy}{dt} = f(y)$, which is plotted below. What can we say about the equilibria of this system?



(a) $y = 0$ is stable, $y = \pm 2$ are unstable.
(b) $y = 0$ is unstable, $y = \pm 2$ are stable.
(c) $y = -2, 0$ are stable, $y = 2$ is unstable.
(d) $y = -2$ is unstable, $y = 0, 2$ are unstable
(e) None of the above

44. **True or False** A differential equation could have infinitely many equilibria.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

45. **True or False** A differential equation could have infinitely many equilibria over a finite range.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

46. Consider the differential equation $\frac{df}{dx} = af + b$, where a and b are non-negative parameters. This equation would have no equilibrium if

(a) $a = 0$

(b) $b = 0$

(c) $a = 1$

(d) More than one of the above

47. What is the equilibrium value of $\frac{dg}{dz} = -\frac{1}{2}g + 3e^z$?

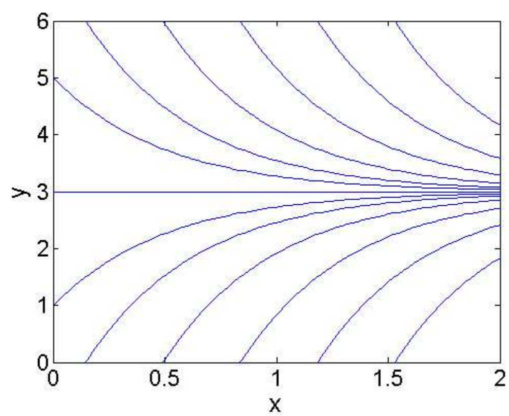
(a) This system is at equilibrium when $g = 6e^z$.

(b) This system is at equlibrium when $z = \ln\left(\frac{g}{6}\right)$.

(c) Both a and b are true.

(d) This equation has no equlibrium.

48. The figure below plots several functions which all solve the differential equation $y' = ay + b$. What could be the values of a and b?



(a) $a = 1$, $b = 3$

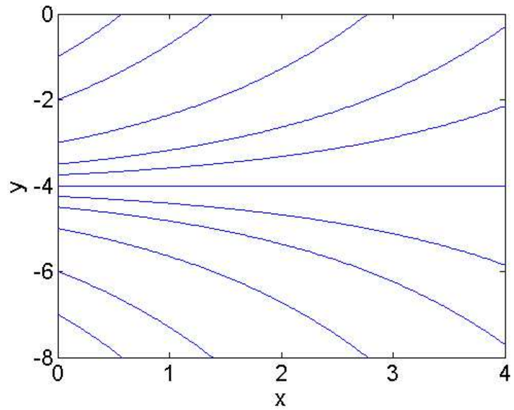
(b) $a = 2$, $b = -6$

(c) $a = -1$, $b = -3$

(d) $a = -2$, $b = 6$

(e) $b = 3$ but a is not easy to tell

49. The figure below plots several functions which all solve the differential equation $\frac{dy}{dx} = ay + b$. What could be the values of a and b?



(a) $a = 0.5$, $b = 2$

(b) $a = 0.5$, $b = -2$

(c) $a = -0.5$, $b = 2$

(d) $a = -0.5$, $b = -2$

(e) None of the above are possible.

Separation of Variables

50. Which of the following DE's is/are separable?

(a) $dy/dx = xy$

(b) $dy/dx = x + y$

(c) $dy/dx = \cos(xy)$

(d) Both (a) and (b)

(e) Both (a) and (c)

(f) All of the above

51. Which of the following differential equations is not separable?

(a) $y' = 3\sin x \cos y$

(b) $y' = x^2 + 3y$

(c) $y' = e^{2x+y}$

(d) $y' = 4x + 7$

(e) More than one of the above

52. Which of the following differential equations is not separable?

(a) $\frac{dx}{dt} = xt^2 - 4x$

(b) $\frac{dx}{dt} = 3x^2t^3$

(c) $\frac{dx}{dt} = \sin(2xt)$

(d) $\frac{dx}{dt} = t^4 \ln(5x)$

53. Which of the following differential equations is separable?

(a) $uu' = 2x + u$

(b) $3ux = \sin(u')$

(c) $\frac{2x^3}{5u'+u} = 1$

(d) $e^{2u'x^2} = e^{u^3}$

54. If we separate the variables in the differential equation $3z't = z^2$, what do we get?

(a) $3z^{-2}dz = t^{-1}dt$

(b) $3tdt = z^2dt$

(c) $3z'tdz = z^2dt$

(d) $z = \sqrt{3z't}$

(e) This equation cannot be separated.

55. If we separate the variables in the differential equation $y' = 2y + 3$, what do we get?

(a) $\frac{dy}{2y} = 3dx$

(b) $dy = 2y = 3dx$

(c) $\frac{dy}{y} = 5dx$

(d) $\frac{dy}{2y+3} = dx$

(e) This equation cannot be separated.

56. What is the solution to the differential equation: $\frac{dy}{dx} = 2xy$.

(a) $y = e^{x^2} + C$

(b) $y = Ce^{x^2}$

(c) $y = e^{2x} + C$

(d) $y = Ce^{2x}$

57. The general solution to the equation $dy/dt = ty$ is

(a) $y = t^2/2 + C$
(b) $y = \sqrt{t^2 + C}$
(c) $y = e^{t^2/2} + C$
(d) $y = Ce^{t^2/2}$
(e) Trick question, equation is not separable

58. The general solution to the equation $\dfrac{dR}{dy} + R = 1$ is

(a) $R = 1 - \sqrt{\dfrac{1}{C - y}}$
(b) $R = 1 - Ce^{y}$
(c) $R = 1 - Ce^{-y}$
(d) Trick question, equation is not separable

59. A plant grows at a rate that is proportional to the square root of its height $h(t)$ – use k as the constant of proportionality. If we separate the variables in the differential equation for its growth, what do we get?

(a) $kh^{1/2}dt = dh$
(b) $\sqrt{hdh} = kdt$
(c) $h^{1/2}dh = kdt$
(d) $h^{-1/2}dh = kdt$
(e) None of the above

Existence & Uniqueness

60. Based upon observations, Kate developed the differential equation $\frac{dT}{dt} = -0.08(T - 72)$ to predict the temperature in her vanilla chai tea. In the equation, T represents the temperature of the chai in °F and t is time. Kate has a cup of chai whose initial temperature is 110°F and her friend Nate has a cup of chai whose initial temperature is 120°F. According Kate's model, will there be a point in time when the two cups of chai have exactly the same temperature?

(a) Yes
(b) No

(c) Can't tell with the information given

61. A bucket of water has a hole in the bottom, and so the water is slowly leaking out. The height of the water in the bucket is thus a decreasing function of time $h(t)$ which changes according to the differential equation $h' = -kh^{1/2}$, where k is a positive constant that depends on the size of the hole and the bucket. If we start out a bucket with 25 cm of water in it, then according to this model, will the bucket ever be empty?

   (a) Yes
   (b) No
   (c) Can't tell with the information given

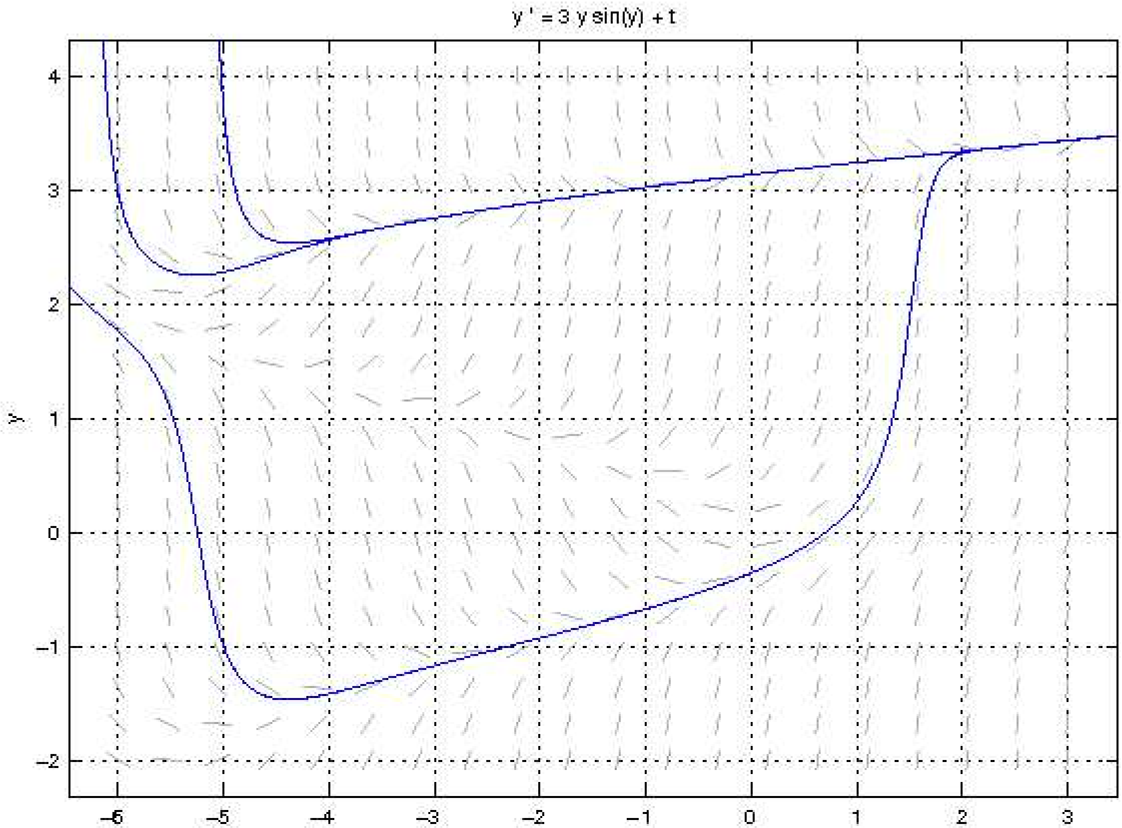
62. A scientist develops the logistic population model $P' = 0.2P(1 - \frac{P}{8.2})$ to describe her research data. This model has an equilibrium value of $P = 8.2$. In this model, from the initial condition $P_0 = 4$, the population never reaches the equilibrium value because:

   (a) you can't have a population of 8.2.
   (b) the population is asymptotically approaching a value of 8.2.
   (c) the population will grow towards infinity.
   (d) the population will drop to 0.

63. A scientist develops the logistic population model $P' = 0.2P(1 - \frac{P}{8})$ to describe her research data. This model has an equilibrium value of $P = 8$. In this model, from the initial condition $P_0 = 4$, the population will never reach the equilibrium value because:

   (a) the population will grow toward infinity.
   (b) the population is asymptotically approaching a value of 8.
   (c) the population will drop to 0.

64. Do the solution trajectories shown below for the for the differential equation $y' = 3y\sin(y) + t$ ever simultaneously reach the same value?



(a) Yes

(b) No

(c) Can't tell with the information given

Chapter 2: Linearity and Nonlinearity

Nonhomogeneous Differential Equations & Undetermined Coefficients

65. Consider the equation $\frac{df}{dx} = 2f + e^{3x}$. When we separate the variables, this equation becomes:

(a) $\frac{1}{2f}df = e^{3x}dx$

(b) $df = 2f + e^{3x}dx$

(c) $\frac{1}{f}df = (2 + e^{3x})dx$

(d) $-2f df = e^{3x}dx$

(e) This equation is not separable

66. $x'(t) + 4x(t) = e^t$ and we want to test the function $x(t) = C_0e^{-4t} + C_1e^t$ to see if it is a solution. What equation is the result?

(a) $(-4C_0e^{-4t} + C_1e^t) + 4C_0e^{-4t} + 4C_1e^t = e^t$
(b) $(C_0e^{-4t} + C_1e^t) + 4C_0e^{-4t} + 4C_1e^t = e^t$
(c) $-4C_0e^{-4t} + 4C_1e^t = e^t$
(d) None of the above

67. We are testing the function $f(x) = C_0e^{3x}$ as a possible solution to a differential equation. After we substitute the function and its derivative into the differential equation we get: $3C_0e^{3x} = -2C_0e^{3x} + 4e^{3x}$. What was the differential equation?

(a) $f' = -2f + \frac{4}{C_0}f$
(b) $f' = -2f + 4e^{3x}$
(c) $3f = -2f + 4e^{3x}$
(d) $3C_0e^{3x} = -2f + 4e^{3x}$
(e) None of the above.

68. We are testing the function $f(x) = C_0e^{2x} + C_1e^{-2x}$ as a possible solution to a differential equation. After we substitute the function and its derivative into the differential equation we get: $2C_0e^{2x} - 2C_1e^{-2x} = -2(C_0e^{2x} + C_1e^{-2x}) + 3e^{2x}$. What value of C_0 will allow this function to work?

(a) $C_0 = \frac{3}{4}$
(b) $C_0 = \frac{3}{2}$
(c) $C_0 = 3$
(d) $C_0 = 2$
(e) Any value of C_0 will work.
(f) No value of C_0 will work.

69. We are testing the function $f(x) = C_0e^{2x} + C_1e^{-2x}$ as a possible solution to a differential equation. After we substitute the function and its derivative into the differential equation we get: $2C_0e^{2x} - 2C_1e^{-2x} = -2(C_0e^{2x} + C_1e^{-2x}) + 3e^{2x}$. What value of C_1 will allow this function to work?

(a) $C_1 = \frac{3}{4}$
(b) $C_1 = \frac{3}{2}$
(c) $C_1 = 3$

(d) $C_1 = 2$

(e) Any value of C_1 will work.

(f) No value of C_1 will work.

70. When we have $y' = 7y + 2x$ we should conjecture $y = C_0 e^{7x} + C_1 x + C_2$. Why do we add the C_2?

    (a) Because the $7y$ becomes a constant 7 when we take the derivative and we need a term to cancel this out.

    (b) Because when we take the derivative of $C_1 x$ we get a constant C_1 and we need a term to cancel this out.

    (c) Because this will allow us to match different initial conditions.

    (d) This does not affect the equation because it goes away when we take the derivative.

71. We have the equation $y' = 2y + \sin 3t$. What should be our conjecture?

    (a) $y = C_0 e^{2t} + \sin 3t$

    (b) $y = C_0 e^{2t} + \sin 3t + \cos 3t$

    (c) $y = C_0 e^{2t} + C_1 \sin 3t$

    (d) $y = C_0 e^{2t} + C_1 \sin 3t + C_2 \cos 3t$

    (e) $y = C_0 e^{2t} + C_1 e^{-2t} + C_2 \sin 3t + C_3 \cos 3t$

    (f) None of the above

72. Consider $\frac{dg}{dz} = ag + b \cos cz$, where a, b, and c are all positive parameters. What will be the long term behavior of this system?

    (a) It will grow exponentially.

    (b) It will converge to an equilibrium.

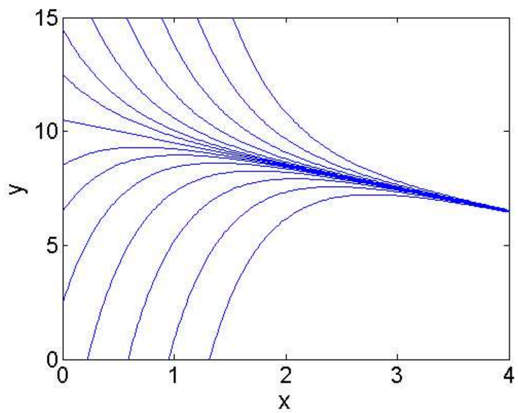
    (c) It will oscillate.

    (d) Different behaviors are possible depending on the values of a, b, and c.

73. A bookstore is constantly discarding a certain percentage of its unsold inventory and also receiving new books from its supplier so that the rate of change of the number of books in inventory is $B'(t) = -0.02B + 400 + 0.05t$, where B is the number of books and t is in months. In the long run, what will happen to the number of books in inventory, according to this model?

(a) The number of books will approach zero.
(b) The number of books will approach a stable equilibrium.
(c) The number of books will exponentially diverge from an unstable equilibrium.
(d) The number of books will grow linearly.
(e) None of the above

74. The figure below shows several functions that solve the differential equation $y' = ay + bx + c$. What could be the values of a, b, and c?



(a) $a = 2$, $b = 2$, $c = 20$
(b) $a = -2$, $b = -2$, $c = -20$
(c) $a = 2$, $b = -2$, $c = -20$
(d) $a = -2$, $b = -2$, $c = 20$
(e) $a = -2$, $b = 2$, $c = 20$
(f) Not enough information is given.

75. It is currently 10 degrees outside and your furnace goes out, so the temperature of your house will follow $\frac{dT}{dt} = 0.1(10 - T)$. You find an old heater which will add heat to your house at a rate of $h(t) = 3 + 2\sin 0.1t$ degrees per hour. What should you conjecture as a function to describe the temperature of your house?

(a) $T(t) = Ae^{-0.1t} + B$
(b) $T(t) = A\sin 0.1t + B$
(c) $T(t) = Ae^{-0.1t} + B\sin 0.1t + C$
(d) $T(t) = Ae^{-0.1t} + B\sin 0.1t + C\cos 0.1t + D$
(e) $T(t) = Ae^{0.1t} + Be^{-0.1t} + C\sin 0.1t + D\cos 0.1t + E$
(f) None of the above.

76. Which of the following is not a solution to $y'(t) = 5y + 3t$?

(a) $y = 8e^{5t}$
(b) $y = -\frac{3}{5}t - \frac{3}{25}$
(c) $y = 8e^{5t} - \frac{3}{5}t - \frac{3}{25}$
(d) All are solutions.
(e) More than one of (a) - (c) are not solutions.

Linear Operators

77. Which differential operator is the appropriate one for the differential equation $ty'' + 2y' + ty = e^t$?

(a) $t\frac{d^2}{dt^2} + 2\frac{d}{dt} + t$
(b) $t\frac{d^2}{dt^2} + 2\frac{d}{dt} + t - e^t$
(c) $\frac{d^2}{dt^2}t + 2\frac{d}{dt} + t$
(d) $\frac{d^2}{dt^2} + 2\frac{d}{dt} + 1$
(e) None of the above

78. Let $L = t^2\frac{d^2}{dt^2} + 2t\frac{d}{dt} + 2$ be a differential operator. Evaluate $L[t^3]$.

(a) 0
(b) $14t^3$
(c) $2 + 12t^3$
(d) $6t^3 + 6t^2 + 2$

Integrating Factors

79. If y is a function of t, which of the following is $t\left(y' + \dfrac{1}{t}y\right)$ equivalent to?

(a) $[ty]'$
(b) $\left[\dfrac{1}{t}y\right]'$
(c) ty'

(d) $\frac{1}{t}y'$

(e) None of the above

80. Which of the following is an integrating factor for $y' + 3ty = \sin t$?

(a) $e^{\frac{3}{2}t^2}$

(b) e^3

(c) $e^{\sin t}$

(d) $e^{-\cos t}$

(e) e^{3y}

(f) All of the above

81. Which of the following is an integrating factor for $y' + 2y = 3t$?

(a) e^{2t}

(b) e^{2t+5}

(c) $e^2 e^{2t}$

(d) $7e^{2t}$

(e) All of the above

(f) None of the above

82. Which of the following is an integrating factor for $3y' + 6ty = 8t$?

(a) e^{3t^2}

(b) e^{t^2}

(c) e^6

(d) All of the above

(e) None of the above

(f) This problem cannot be solved with integrating factors.

83. Can integrating factors be used to solve $y' + 2ty = 1$?

(a) Yes. The solution is $y = e^{-t^2} \int e^{t^2} dt$.

(b) No - we cannot evaluate $\int e^{t^2} dt$.

84. The differential equation $y' + 2y = 3t$ is solved using integrating factors. The solution is $y = \frac{3}{2}t - \frac{3}{4} + Ce^{-2t}$. Which of the following statements describes the long-term behavior as $t \to \infty$?

(a) The solutions will approach zero, because $e^{-2t} \to 0$ as $t \to \infty$.

(b) The solutions will grow without bound because $\frac{3}{2}t \to \infty$ as $t \to \infty$.

(c) The long-term behavior depends on the inital condition; we need to know the value of C before we can answer this.

85. Which of the following is $e^{3t}(y' + 2y)$ equivalent to?

(a) $[e^{2t}y]'$

(b) $[e^{3t}y]'$

(c) $e^{3t}y'$

(d) $e^{2t}y'$

(e) None of the above

Exponential Solutions, Growth and Decay

86. A star's brightness is decreasing at a rate equal to 10% of its current brightness per million years. If B_0 is a constant with units of brightness and t is in millions of years, what function could describe the brightness of the star?

(a) $B'(t) = -0.1B(t)$

(b) $B(t) = B_0e^t$

(c) $B(t) = B_0e^{-0.1t}$

(d) $B(t) = B_0e^{0.1t}$

(e) $B(t) = B_0e^{0.9t}$

(f) $B(t) = -0.1B_0t$

87. A small company grows at a rate proportional to its size, so that $c'(t) = kc(t)$. We set $t = 0$ in 1990 when there were 50 employees. In 2005 there were 250 employees. What equation must we solve in order to find the growth constant k?

(a) $50e^{2005k} = 250$

(b) $50e^{15k} = 250$

(c) $250e^{15k} = 50$

(d) $50e^{tk} = 250$

(e) Not enough information is given.

88. What differential equation is solved by the function $f(x) = 0.4e^{2x}$?

(a) $\frac{df}{dx} = 0.4f$

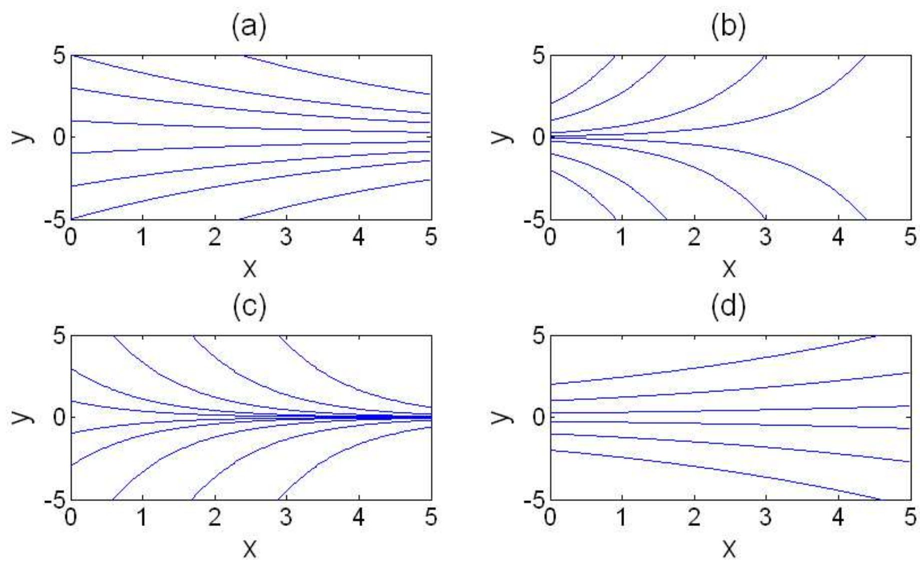
(b) $\frac{df}{dx} = 2f$

(c) $\frac{df}{dx} = 2f + 0.4$

(d) $\frac{df}{dx} = 0.4f + 2$

(e) None of the above.

89. Each of the graphs below show solutions of $y' = k_i y$ for a different k_i. Rank these constants from smallest to largest.



(a) $k_b < k_d < k_a < k_c$

(b) $k_d < k_c < k_b < k_a$

(c) $k_c < k_a < k_d < k_b$

(d) $k_a < k_b < k_c < k_d$

90. The function $f(y)$ solves the differential equation $f' = -0.1f$ and we know that $f(0) > 0$. This means that:

(a) When y increases by 1, f decreases by exactly 10%.

(b) When y increases by 1, f decreases by a little more than 10%.

(c) When y increases by 1, f decreases by a little less than 10%.

(d) Not enough information is given.

91. The function $g(z)$ solves the differential equation $\frac{dg}{dz} = 0.03g$. This means that:

    (a) g is an increasing function that changes by 3% every time z increases by 1.

    (b) g is an increasing function that changes by more than 3% every time z increases by 1.

    (c) g is an increasing function that changes by less than 3% every time z increases by 1.

    (d) g is a decreasing function that changes by more than 3% every time z increases by 1.

    (e) g is a decreasing function that changes by less than 3% every time z increases by 1.

    (f) Not enough information is given.

92. 40 grams of a radioactive element with a half-life of 35 days are put into storage. We solve $y' = -ky$ with $k = 0.0198$ to find a function that describes how the amount of this element will decrease over time. Another facility stores 80 grams of the element and we want to derive a similar function. When solving the differential equation, what value of k should we use?

    (a) $k = 0.0099$

    (b) $k = 0.0198$

    (c) $k = 0.0396$

    (d) None of the above

93. A star's brightness is decreasing at a rate equal to 10% of its current brightness per million years, so $B'(t) = -0.1B(t)$, where t is measured in millions of years. If we want t to be measured in years, how would the differential equation change?

    (a) $B'(t) = -0.1B(t)$

    (b) $B'(t) = -10^5 B(t)$

    (c) $B'(t) = -10^{-6} B(t)$

    (d) $B'(t) = -10^{-7} B(t)$

    (e) None of the above

94. The solution to which of the following will approach $+\infty$ as x becomes very large?

(a) $y' = -2y$, $y(0) = 2$
(b) $y' = 0.1y$, $y(0) = 1$
(c) $y' = 6y$, $y(0) = 0$
(d) $y' = 3y$, $y(0) = -3$
(e) None of the above

95. $y' = -\frac{1}{3}y$ with $y(0) = 2$. As x becomes large, the solution will

(a) diverge to $+\infty$.
(b) diverge to $-\infty$.
(c) approach 0 from above.
(d) approach 0 from below.
(e) do none of the above.

96. Suppose H is the temperature of a hot object placed into a room whose temperature is 70 degrees, and t represents time. Suppose k is a positive number. Which of the following differential equations best corresponds to Newton's Law of Cooling?

(a) $dH/dt = -kH$
(b) $dH/dt = k(H - 70)$
(c) $dH/dt = -k(H - 70)$
(d) $dH/dt = -k(70 - H)$
(e) $dH/dt = -kH(H - 70)$

97. Suppose H is the temperature of a hot object placed into a room whose temperature is 70 degrees.The function H giving the object's temperature as a function of time is most likely

(a) Increasing, concave up
(b) Increasing, concave down
(c) Decreasing, concave up
(d) Decreasing, concave down

98. Suppose H is the temperature of a hot object placed into a room whose temperature is 70 degrees, and t represents time. Then $\lim\limits_{t\to\infty} H$ should equal approximately

(a) $-\infty$
(b) 0
(c) 32
(d) 70
(e) Whatever the difference is between the object's initial temperature and 70

Mixing Models

99. The differential equation for a mixing problem is $x' + 0.08x = 4$, where x is the amount of dissolved substance, in pounds, and time is measured in minutes. What are the units of 4?

    (a) pounds
    (b) minutes
    (c) pounds/minute
    (d) minutes/pound
    (e) None of the above

100. The differential equation for a mixing problem is $x' + 0.08x = 4$, where x is the amount of dissolved substance, in pounds, and time is measured in minutes. What are the units of 0.08?

    (a) pounds
    (b) minutes
    (c) per pound
    (d) per minute
    (e) None of the above

101. The differential equation for a mixing problem is $x' + 0.08x = 4$, where x is the amount of dissolved substance, in pounds, and time is measured in minutes. What is the equilibrium value for this model?

    (a) 0.08
    (b) 50
    (c) 0
    (d) 0.02

(e) 4

(f) None of the above

102. In a mixing model where x' has units of pounds per minute, the equilibirum value is 80. Which of the following is a correct interpretation of the equilibrium?

(a) In the long-run, there will be 80 pounds of contaminant in the system.

(b) After 80 minutes, the mixture will stabilize.

(c) The rate in is equal to the rate out when the concentration of contaminant is 80 pounds per gallon.

(d) The rate in is equal to the rate out when the amount of contaminant is 80 pounds.

(e) None of the above

103. A tank initially contains 60 gallons of pure water. A solution containing 3 pounds/gallon of salt is pumped into the tank at a rate of 2 gallons/minute. The mixture is stirred constantly and flows out at a rate of 2 gallons per minute. If $x(t)$ is the amount of salt in the tank at time t, which initial value problem represents this scenario?

(a) $x'(t) = 2 - 2$, with $x(0) = 60$

(b) $x'(t) = 6 - \frac{x}{30}$, with $x(0) = 0$

(c) $x'(t) = 3x - 2$, with $x(0) = 60$

(d) $x'(t) = 6 - \frac{x}{60}$, with $x(0) = 0$

(e) None of the above

104. The solution to a mixing problem is

$$x(t) = -0.01(100 - t)^2 + (100 - t),$$

where $x(t)$ is the amount of a contaminant in a tank of water. What is the long-term behavior of this solution?

(a) The amount of contaminant will reach a steady-state of 100 pounds.

(b) The amount of contaminant will increase forever.

(c) The amount of contaminant will approach zero.

(d) The tank will run out of water.

105. Tank A initially contains 30 gallons of pure water, and tank B initially contains 40 gallons of pure water. A solution containing 2 pounds/gallon of salt is pumped into tank A at a rate of 1.5 gallons/minute. The mixture in tank A is stirred constantly and flows into tank B at a rate of 1.5 gallons/minute. The mixture in tank B is also stirred constantly, and tank B drains at a rate of 1.5 gallons/minute. If $A(t)$ is the amount of salt in the tank A at time t and $B(t)$ is the amount of salt in tank B at time t, which initial value problem represents this scenario?

(a)

$$\begin{aligned} A'(t) &= 1.5 - \frac{A}{30} \qquad & A(0) = 0 \\ B'(t) &= 1.5 - \frac{B}{40} \qquad & B(0) = 0 \end{aligned}$$

(b)

$$\begin{aligned} A'(t) &= 1.5 - \frac{A}{30} \qquad & A(0) = 0 \\ B'(t) &= \frac{A}{30} - \frac{B}{40} \qquad & B(0) = 0 \end{aligned}$$

(c)

$$\begin{aligned} A'(t) &= 3 - \frac{A}{30} \qquad & A(0) = 0 \\ B'(t) &= \frac{A}{30} - \frac{B}{40} \qquad & B(0) = 0 \end{aligned}$$

(d)

$$\begin{aligned} A'(t) &= 3 - \frac{A}{20} \qquad & A(0) = 0 \\ B'(t) &= \frac{A}{20} - \frac{3B}{80} \qquad & B(0) = 0 \end{aligned}$$

(e) None of the above

106. Medication flows from the GI tract into the bloodstream. Suppose that A units of an antihistamine are present in the GI tract at time 0 and that the medication moves from the GI tract into the blood at a rate proportional to the amount in the GI tract, x. Assume that no further medication enters the GI tract, and that the kidneys and liver clear the medication from the blood at a rate proportional to the amount currently in the blood, y. If k_1 and k_2 are positive constants, which initial value problem models this scenario?

(a)

$$\begin{aligned}\frac{dx}{dt} &= k_1x - k_2y \qquad x(0) = A\\ \frac{dy}{dt} &= k_2y \qquad y(0) = 0\end{aligned}$$

(b)

$$\begin{aligned}\frac{dx}{dt} &= k_1x - k_2y \qquad x(0) = 0\\ \frac{dy}{dt} &= k_2y \qquad y(0) = 0\end{aligned}$$

(c)

$$\begin{aligned}\frac{dx}{dt} &= -k_1x \qquad x(0) = A\\ \frac{dy}{dt} &= k_1x - k_2y \qquad y(0) = 0\end{aligned}$$

(d)

$$\begin{aligned}\frac{dx}{dt} &= k_1x - k_2y \qquad x(0) = 0\\ \frac{dy}{dt} &= k_1x + k_2y \qquad y(0) = 0\end{aligned}$$

107. Referring to the antihistamine model developed in the previous question, if time is measured in hours and the quantity of antihistamine is measured in milligrams, what are the units of k_1?

   (a) hours
   (b) hours per milligram
   (c) milligrams per hour
   (d) 1/hours
   (e) milligrams
   (f) None of the above

108. Referring to the antihistamine model discussed in the previous questions, what is the effect of increasing k_2?

   (a) The blood will be cleaned faster.
   (b) Antihistamine will be removed from the GI tract at a faster rate.

(c) It will take longer for the antihistamine to move from the GI tract into the blood.

(d) Antihistamine will accumulate in the blood at a faster rate and the patient will end up with an overdose.

(e) None of the above

109. Referring to the antihistamine model discussed in the previous questions, describe how the amount of antihistamine in the blood changes with time.

(a) It decreases and asymptotically approaches zero.

(b) It levels off at some nonzero value.

(c) It increases indefinitely.

(d) It increases, reaches a peak, and then decreases, asymptotically approaching zero.

(e) None of the above

First Order Linear Models

110. Water from a thunderstorm flows into a reservoir at a rate given by the function $g(t) = 250e^{-0.1t}$, where g is in gallons per day, and t is in days. The water in the reservoir evaporates at a rate of 2.25% per day. What equation could describe this scenario?

(a) $f'(t) = -0.0225f + 250e^{-0.1t}$

(b) $f'(t) = -0.0225(250e^{-0.1t})$

(c) $f'(t) = 0.9775f + 250e^{-0.1t}$

(d) None of the above

111. The state of ripeness of a banana is described by the differential equation $R'(t) = 0.05(2 - R)$ with $R = 0$ corresponding to a completely green banana and $R = 1$ a perfectly ripe banana. If all bananas start completely green, what value of R describes the state of a completely black, overripe banana?

(a) $R = 0.05$

(b) $R = \frac{1}{2}$

(c) $R = 1$

(d) $R = 2$

(e) $R = 4$

(f) None of the above.

112. The evolution of the temperature T of a hot cup of coffee cooling off in a room is described by $\frac{dT}{dt} = -0.01T + 0.6$, where T is in °F and t is in hours. What is the temperature of the room?

(a) 0.6
(b) -0.01
(c) 60
(d) 0.006
(e) 30
(f) none of the above

113. The evolution of the temperature of a hot cup of coffee cooling off in a room is described by $\frac{dT}{dt} = -0.01(T-60)$, where T is in °F and t is in hours. Next, we add a small heater to the coffee which adds heat at a rate of 0.1 °F per hour. What happens?

(a) There is no equilibrium, so the coffee gets hotter and hotter.
(b) The coffee reaches an equilibrium temperature of 60°F.
(c) The coffee reaches an equilibrium temperature of 70°F.
(d) The equilibrium temperature becomes unstable.
(e) None of the above

114. A drug is being administered intravenously into a patient at a certain rate d and is breaking down at a certain fractional rate $k > 0$. If $c(t)$ represents the concentration of the drug in the bloodstream, which differential equation represents this scenario?

(a) $\frac{dc}{dt} = -k + d$
(b) $\frac{dc}{dt} = -kc + d$
(c) $\frac{dc}{dt} = kc + d$
(d) $\frac{dc}{dt} = c(d - k)$
(e) None of the above

115. A drug is being administered intravenously into a patient. The drug is flowing into the bloodstream at a rate of 50 mg/hr. The rate at which the drug breaks down is proportional to the total amount of the drug, and when there is a total of 1000 mg of the drug in the patient, the drug breaks down at a rate of 300 mg/hr. If y is the number of milligrams of drug in the bloodstream at time t, what differential equation would describe the evolution of the amount of the drug in the patient?

(a) $y' = -0.3y + 50$

(b) $y' = -0.3t + 50$

(c) $y' = 0.7y + 50$

(d) None of the above

116. The amount of a drug in the bloodstream follows the differential equation $c' = -kc+d$, where d is the rate it is being added intravenously and k is the fractional rate at which it breaks down. If the initial concentration is given by a value $c(0) > d/k$, then what will happen?

(a) This equation predicts that the concentration of the drug will be negative, which is impossible.

(b) The concentration of the drug will decrease until there is none left.

(c) This means that the concentration of the drug will get smaller, until it reaches the level $c = d/k$, where it will stay.

(d) This concentration of the drug will approach but never reach the level d/k.

(e) Because $c(0) > d/k$ this means that the concentration of the drug will increase, so the dose d should be reduced.

117. The amount of a drug in the bloodstream follows the differential equation $c' = -kc+d$, where d is the rate it is being added intravenously and k is the fractional rate at which is breaks down. If we double the rate at which the drug flows in, how will this change the equilibrium value?

(a) It will be double the old value.

(b) It will be greater than the old, but not quite doubled.

(c) It will be more than doubled.

(d) It will be the same.

(e) Not enough information is given.

118. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation $V_{bat} = \frac{Q}{C} + IR$. Here V_{bat} is the voltage produced by the battery, and the constants C and R give the capacitance and resistance respectively. $Q(t)$ is the charge on the capacitor and $I(t) = \frac{dQ}{dt}$ is the current flowing through the circuit. What is the equilibrium charge on the capacitor?

(a) $Q_e = V_{bat}C$

(b) $Q_e = V_{bat}/R$

(c) $Q_e = 0$

(d) Not enough information is given.

119. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation $V_{bat} = \frac{Q}{C} + IR$. Here V_{bat} is the voltage produced by the battery, and the constants C and R give the capacitance and resistance respectively. $Q(t)$ is the charge on the capacitor and $I(t) = \frac{dQ}{dt}$ is the current flowing through the circuit. Which of the following functions could describe the charge on the capacitor $Q(t)$?

(a) $Q(t) = 5e^{-t/RC}$

(b) $Q(t) = 4e^{-RCt} + V_{bat}C$

(c) $Q(t) = 3e^{-t/RC} - V_{bat}C$

(d) $Q(t) = -6e^{-t/RC} + V_{bat}C$

(e) None of the above

120. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation $V_{bat} = \frac{Q}{C} + IR$. Here V_{bat} is the voltage produced by the battery, and the constants C and R give the capacitance and resistance respectively. $Q(t)$ is the charge on the capacitor and $I(t) = \frac{dQ}{dt}$ is the current flowing through the circuit. Which of the following functions could describe the current flowing through the circuit $I(t)$?

(a) $I(t) = 5e^{-t/RC}$

(b) $I(t) = 4e^{-RCt} + V_{bat}C$

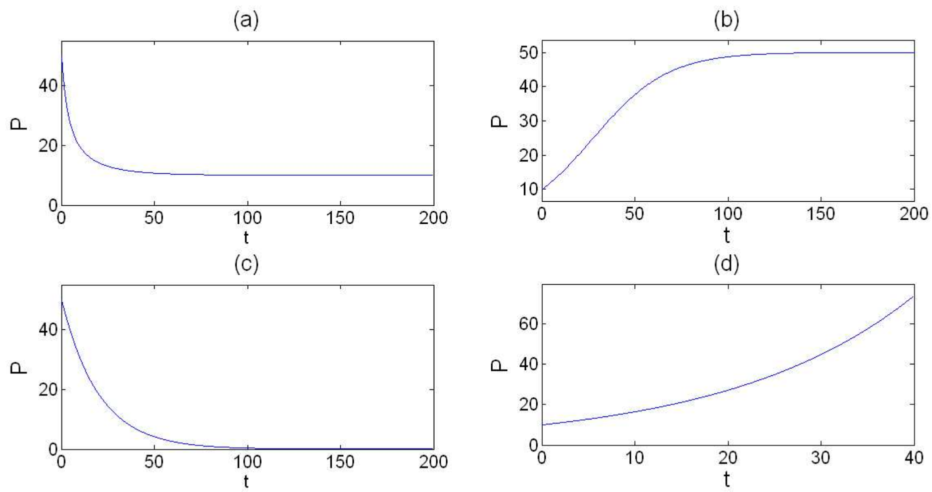
(c) $I(t) = 3e^{-t/RC} - V_{bat}C$

(d) $I(t) = -6e^{-t/RC} + V_{bat}C$

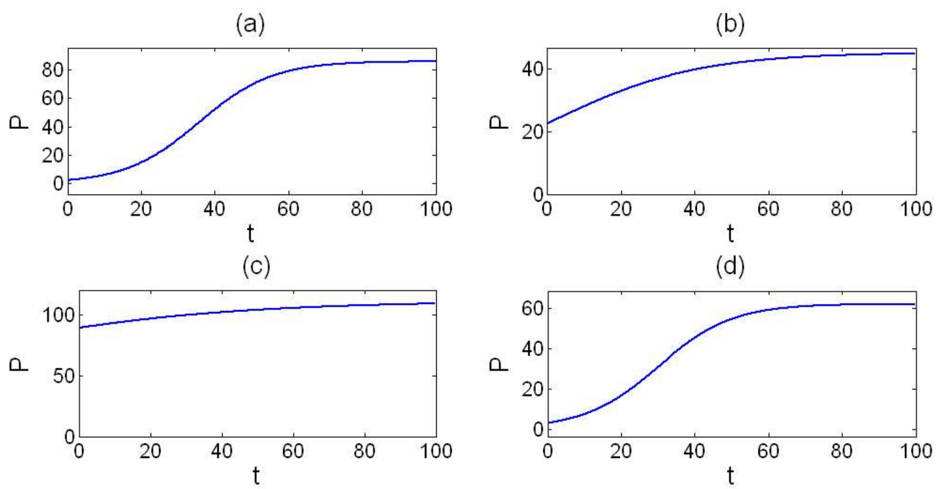
(e) None of the above

Logistic Models

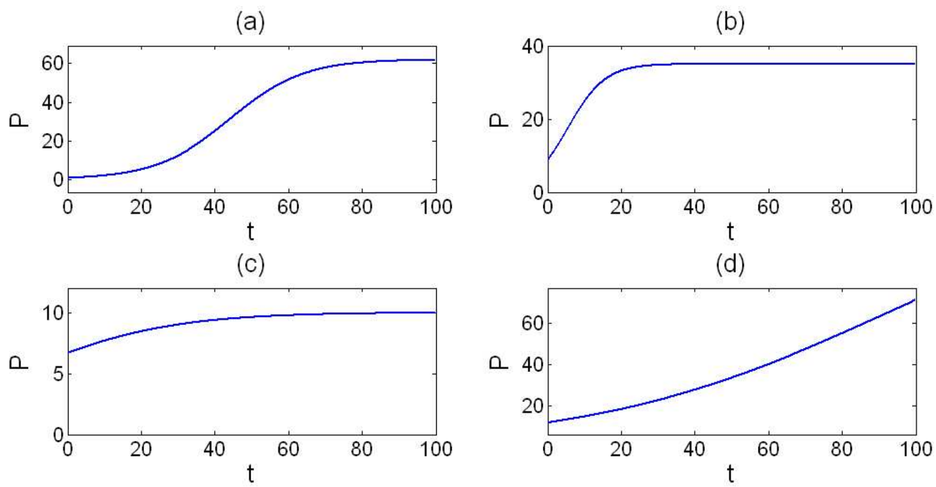
121. Consider the function $P = \frac{L}{1+Ae^{-kt}}$, where $A = (L - P_0)/P_0$. Suppose that $P_0 = 10$, $L = 50$, and $k = 0.05$, which of the following could be a graph of this function?



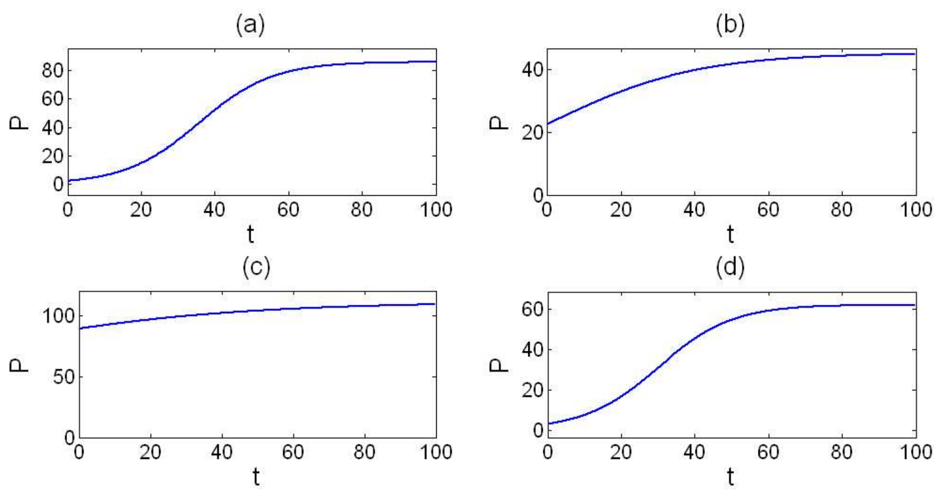
122. The following graphs all plot the function $P = \frac{L}{1+Ae^{-kt}}$. The function plotted in which graph has the largest value of L?



123. The following graphs all plot the function $P = \frac{L}{1+Ae^{-kt}}$. The function plotted in which graph has the largest value of k?



124. The following graphs all plot the function $P = \frac{L}{1+Ae^{-kt}}$. The function plotted in which graph has the largest value of A?



125. Consider the differential equation $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$, called the logistic equation. What are the equilibria of this system?

i. $k = 0$ is a stable equilibrium.
ii. $L = 0$ is an unstable equilibrium.
iii. $P = L$ is a stable equilibrium.
iv. $P = 0$ is an unstable equilibrium.
v. $P = L$ is an unstable equilibrium.
vi $P = 0$ is a stable equilibrium.

(a) i
(b) ii
(c) Both iii and v

(d) Both v and vi

(e) Both iv and v

(f) Both iii and iv

126. The population of rainbow trout in a river system is modeled by the differential equation $P' = 0.2P - 4 \times 10^{-5}P^2$. What is the maximum number of trout that the river system could support?

(a) 4×10^5 trout

(b) 4,000 trout

(c) 5,000 trout

(d) 25,000 trout

(e) Not enough information is given

127. The solution to the logistic equation $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$ is $P = \frac{L}{1+Ae^{-kt}}$, where $A = (L - P_0)/P_0$. If we are modeling a herd of elk, with an initial population of 50, in a region with a carrying capacity of 300, and knowing that the exponential growth rate of an elk population is 0.07, which function would describe our elk population as a function of time?

(a) $P(t) = \frac{300}{1+5e^{-0.07t}}$

(b) $P(t) = \frac{50}{1+\frac{5}{6}e^{0.07t}}$

(c) $P(t) = \frac{300}{1+\frac{6}{5}e^{0.07t}}$

(d) $P(t) = \frac{300}{1+\frac{1}{6}e^{-0.07t}}$

128. The population of mice on a farm is modeled by the differential equation $\frac{3000}{P}\frac{dP}{dt} = 200 - P$. If we know that today there are 60 mice on the farm, what function will describe how the mouse population will develop in the future?

(a) $P = \frac{200}{1+\frac{7}{3}e^{-t/15}}$

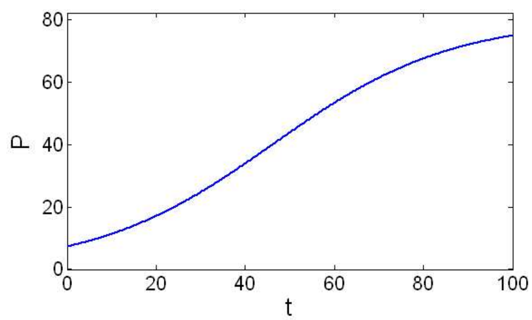
(b) $P = \frac{200}{1+\frac{7}{3}e^{-200t}}$

(c) $P = \frac{3000}{1+49e^{-t/15}}$

(d) $P = \frac{3000}{1+49e^{-t/20}}$

(e) None of the above

129. The function plotted below could be a solution to which of the following differential equations?



(a) $\frac{dP}{dt} = -0.05P\left(1 - \frac{P}{80}\right)$

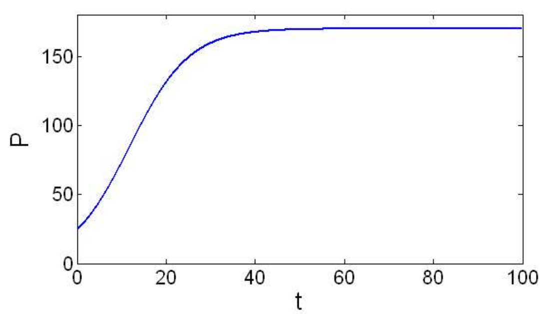
(b) $P = \frac{P'}{\frac{1}{20} - 6.25\times10^{-4}P}$

(c) $40\frac{P'}{P^2} + \frac{1}{40} = \frac{2}{P}$

(d) $-20\frac{dP}{dt} + P = \frac{P^2}{80}$

(e) All of the above

130. The function plotted below could be a solution of which of the following?



(a) $\frac{dP}{dt} = 0.15P\left(1 - \frac{P}{170}\right)$

(b) $\frac{dP}{dt} = 0.15P\left(1 - \frac{P}{240}\right)$

(c) $\frac{dP}{dt} = 0.05P\left(1 - \frac{P}{170}\right)$

(d) $\frac{dP}{dt} = 0.05P\left(1 - \frac{P}{240}\right)$

(e) None of the above

Bifurcations

131. How many equilibria does the differential equation $y' = y^2 + a$ have?

(a) Zero

(b) One

(c) Two

(d) Three

(e) Not enough information is given.

132. A *bifurcation* occurs if the number of equilibria of a system changes when we change the value of a parameter. For the differential equation $\frac{df}{dx} = bf^2 - 2$, a bifurcation occurs at what value of b?

(a) $b = 0$

(b) $b = 2$

(c) $b = -2$

(d) $b = 2/f^2$

(e) Not enough information is given.

133. $x'(t) = \frac{1}{2}x^2 + bx + 8$. If $b = 5$ what are the equilibria of the system?

(a) $x = -5$

(b) $x = -3, 3$

(c) $x = -8, -2$

(d) No equilibria exist and all solutions are increasing.

(e) No equilibria exist and all solutions are decreasing.

134. $x'(t) = \frac{1}{2}x^2 + bx + 8$. If $b = 2$ what are the equilibria of the system?

(a) $x = -8$

(b) $x = -2 \pm \sqrt{12}$

(c) $x = 2$

(d) No equilibria exist and all solutions are increasing.

(e) No equilibria exist and all solutions are decreasing.

135. $x'(t) = \frac{1}{2}x^2 + bx + 8$. A bifurcation occurs where?

(a) $x = -b \pm \sqrt{b^2 - 16}$

(b) $b = 0$

(c) $b = \frac{1}{2}$

(d) $b = 4$

(e) $b = 8$

(f) Not enough information is given.

136. $\frac{dg}{dt} = g^3 + cg$. How many equilibria does this system have?

(a) Two

(b) One or two

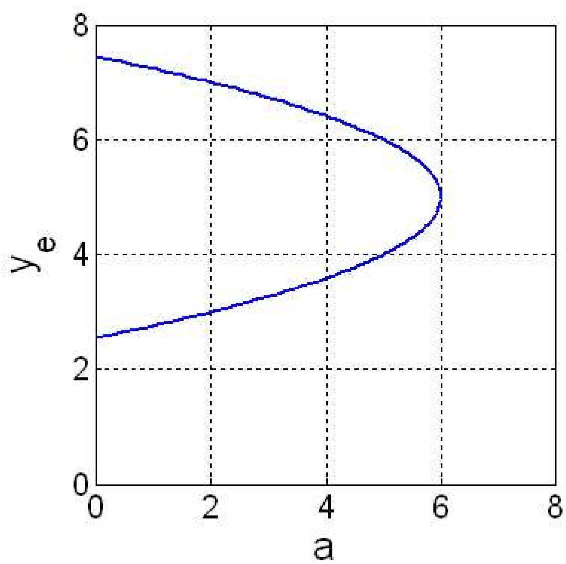
(c) One or three

(d) Two or three

(e) Three

(f) Not enough information is given

137. A *bifurcation diagram* plots a system's equilibria on the y axis and the value of a parameter on the x axis. Consider the bifurcation diagram below. When our parameter is $a = 5$, what are the equilibria of the system?



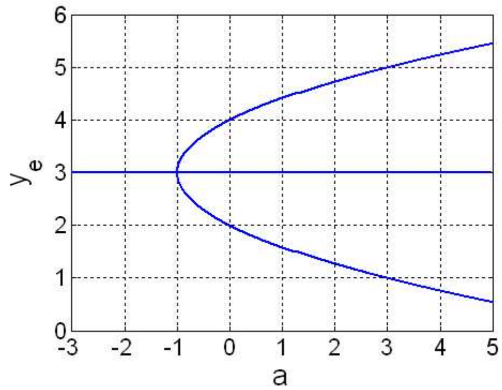
(a) $y = 0$ and $y = 3$

(b) $y = 5$

(c) $y = 4$ and $y = 6$

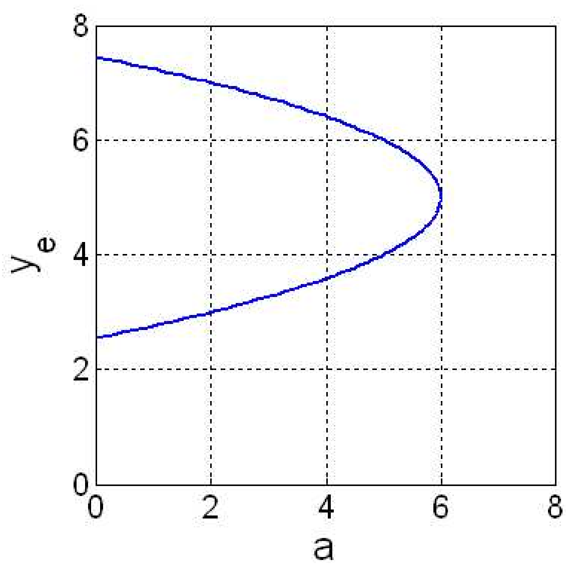
(d) Not enough information is given.

138. Consider the bifurcation diagram below. If our system has equilibria at $y = 1$, $y = 3$ and $y = 5$ what is the value of the parameter a?



(a) $a = -1$
(b) $a = 0$
(c) $a = 1$
(d) $a = 3$
(e) $a = 5$
(f) Not enough information is given.

139. Consider the bifurcation diagram below. At what value of a does the system have a bifurcation?



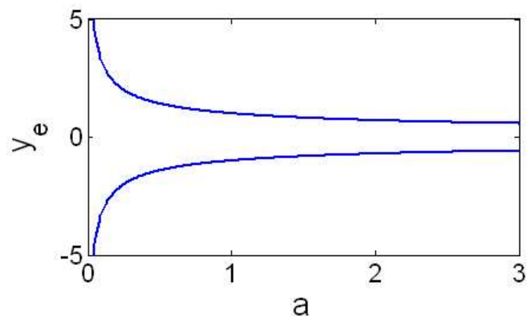
(a) $a = 0$
(b) $a = 2$
(c) $a = 4$

(d) $a = 6$

(e) $a = 8$

(f) Not enough information is given.

140. Which of the following differential equations is represented by the bifurcation diagram below?



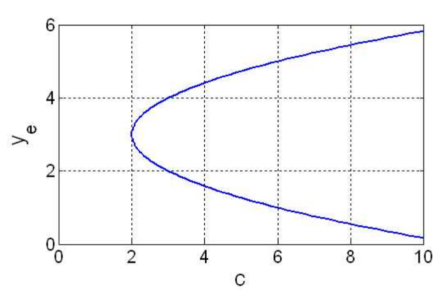
(a) $y' = y^2 + a$

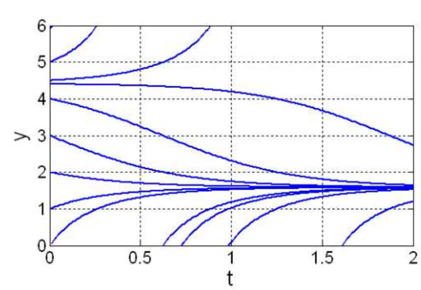
(b) $y' = ay^2 - 1$

(c) $y' = ay$

(d) $y' = y^2 + ay + 2$

141. The figure on the left is a bifurcation diagram, and the figure on the right plots several solution functions of this system for one specific value of the parameter. The figure on the right corresponds to what value of the parameter?





(a) $c = 0$

(b) $c = 2$

(c) $c = 4$

(d) $c = 6$

(e) $c = 8$

Modeling with Systems

142. In the predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= ax - \frac{ax^2}{N} - bxy \\ \frac{dy}{dt} &= cy + kxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, $N > 0$, and $k > 0$,

which variable represents the predator population?

(a) x

(b) $\frac{dx}{dt}$

(c) y

(d) $\frac{dy}{dt}$

143. In which of the following predator - prey population models does the prey have the highest intrinsic reproduction rate?

(a)

$$\begin{aligned}P' &= 2P - 3Q * P \\ Q' &= -Q + 1/2Q * P\end{aligned}$$

(b)

$$\begin{aligned}P' &= P(1 - 4Q) \\ Q' &= Q(-2 + 3P)\end{aligned}$$

(c)

$$\begin{aligned}P' &= P(3 - 2Q) \\ Q' &= Q(-1 + P)\end{aligned}$$

(d)

$$\begin{aligned}P' &= 4P(1/2 - Q) \\ Q' &= Q(-1.5 + 2P)\end{aligned}$$

144. For which of the following predator - prey population models is the predator most successful at catching prey?

(a)

$$\begin{aligned}\frac{dx}{dt} &= 2x - 3x * y \\ \frac{dy}{dt} &= -y + 1/2x * y\end{aligned}$$

(b)

$$\begin{aligned}\frac{dx}{dt} &= x(1 - 4y) \\ \frac{dy}{dt} &= y(-2 + 3x)\end{aligned}$$

(c)

$$\begin{aligned}\frac{dx}{dt} &= x(3 - 2y) \\ \frac{dy}{dt} &= y(-1 + x)\end{aligned}$$

(d)

$$\begin{aligned}\frac{dx}{dt} &= 4x(1/2 - y) \\ \frac{dy}{dt} &= 2y(-1/2 + x)\end{aligned}$$

145. In this predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= -ax + bxy \\ \frac{dy}{dt} &= cy - dxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, and $d > 0$,

does the prey have limits to its population other than that imposed by the predator?

(a) Yes

(b) No
(c) Can not tell

146. In this predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= ax - \frac{ax^2}{N} - bxy\\ \frac{dy}{dt} &= cy + kxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, and $k > 0$,

if the prey becomes extinct, will the predator survive?

(a) Yes
(b) No
(c) Can not tell

147. In this predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= ax - \frac{ax^2}{N} - bxy\\ \frac{dy}{dt} &= cy + kxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, $N > 0$, and $k > 0$,

are there any limits on the prey's population other than the predator?

(a) Yes
(b) No
(c) Can not tell

148. On Komodo Island we have three species: Komodo dragons (K), deer (D), and a variety of plant (P). The dragons eat the deer and the deer eat the plant. Which of the following systems of differential equations could represent this scenario?

(a)

$$\begin{aligned}K' &= aK - bKD\\ D' &= cD + dKD - eDP\\ P' &= -fP + gDP\end{aligned}$$

(b)

$$
\begin{aligned}
K' &= -aK + bKD \\
D' &= -cD - dKD + eDP \\
P' &= fP - gDP
\end{aligned}
$$

(c)

$$
\begin{aligned}
K' &= aK - bKD + KP \\
D' &= cD + dKD - eDP \\
P' &= -fP + gDP - hKP
\end{aligned}
$$

(d)

$$
\begin{aligned}
K' &= -aK + bKD - KP \\
D' &= -cD - dKD + eDP \\
P' &= fP - gDP + hKP
\end{aligned}
$$

149. In the two species population model

$$
\begin{aligned}
R' &= 2R - bFR \\
F' &= -F + 2FR
\end{aligned}
$$

for what value of the parameter b will the system have a stable equilibrium?

(a) $b < 0$
(b) $b = 0$
(c) $b > 0$
(d) For no value of b

150. Two forces are fighting one another. x and y are the number of soldiers in each force. Let a and b be the offensive fighting capacities of x and y, respectively. Assume that forces are lost only to combat, and no reinforcements are brought in. What system represents this scenario?

(a)

$$
\begin{aligned}
\frac{dx}{dt} &= -ay \\
\frac{dy}{dt} &= -bx
\end{aligned}
$$

(b)

$$\frac{dx}{dt} = -by$$
$$\frac{dy}{dt} = -ax$$

(c)

$$\frac{dx}{dt} = y - a$$
$$\frac{dy}{dt} = x - b$$

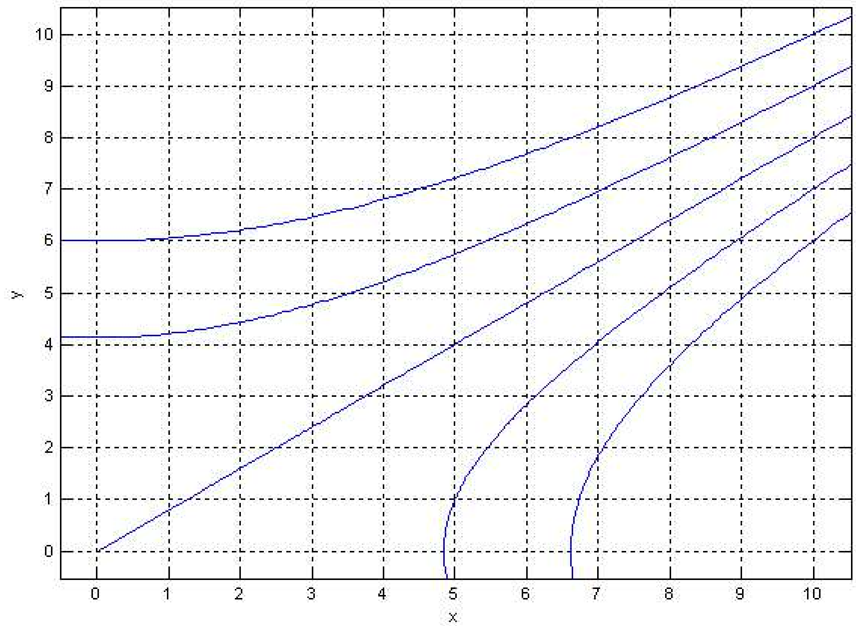
(d)

$$\frac{dx}{dt} = y - b$$
$$\frac{dy}{dt} = x - a$$

151. Two forces, x and y, are fighting one another. Let a and b be the fighting efficiences of x and y, respectively. Assume that forces are lost only to combat, and no reinforcements are brought in. How does the size of the y army change with respect to the size of the x army?

    (a) $\frac{dy}{dx} = \frac{ax}{by}$
    (b) $\frac{dy}{dx} = \frac{x}{y}$
    (c) $\frac{dy}{dx} = \frac{y}{x}$
    (d) $\frac{dy}{dx} = -by - ax$

152. Two forces, x and y, are fighting one another. Assume that forces are lost only to combat, and no reinforcements are brought in. Based on the phase plane below, if $x(0) = 10$ and $y(0) = 7$, who wins?



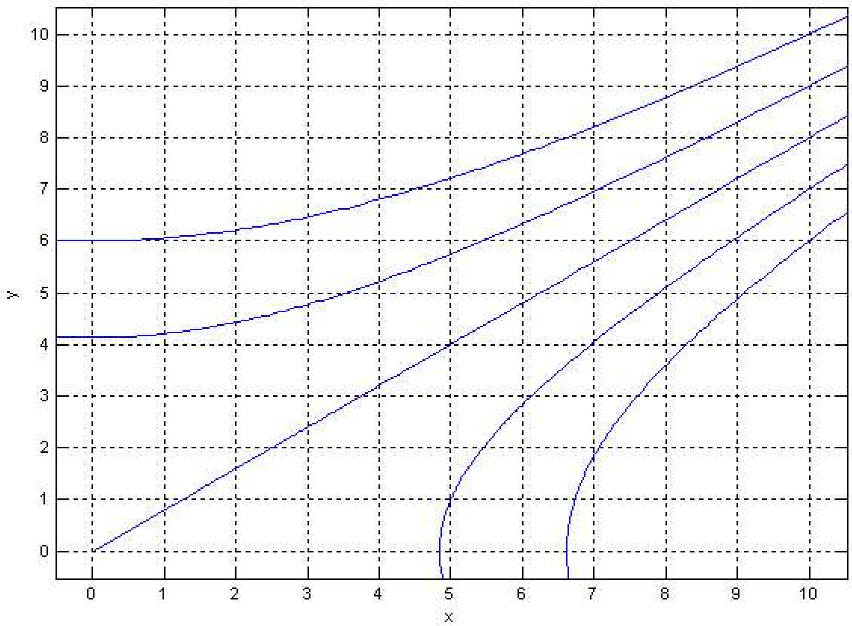
(a) x wins

(b) y wins

(c) They tie.

(d) Neither wins - both armies grow, and the battles escalate forever.

153. Two forces, x and y, are fighting one another. Assume that forces are lost only to combat, and no reinforcements are brought in. Based on the phase plane below, which force has a greater offensive fighting efficiency?



(a) x has the greater fighting efficiency.
(b) y has the greater fighting efficiency.
(c) They have the same fighting efficiencies.

154. Two forces, x and y, are fighting one another. Assume that forces are lost only to combat, and no reinforcements are brought in. You are the x-force, and you want to improve your chance of winning. Assuming that it would be possible, would you rather double your fighting efficiency or double your number of soldiers?

(a) Double the fighting efficiency
(b) Double the number of soldiers
(c) These would both have the same effect

Phase Portraits and Vector Fields of Systems

155. If we were graphing a vector field in the phase plane of the linear system $Y' = \begin{bmatrix} -4 & 2 \\ 2 & 4 \end{bmatrix} Y$, what slope would we graph when $y_1 = 1$ and $y_2 = 2$?

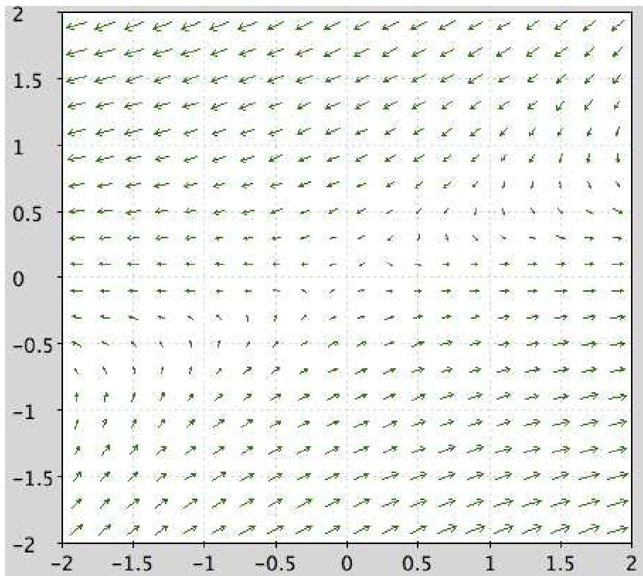
(a) 0

(b) ∞ (vertical)

(c) 1

(d) None of the above

156. Which linear system matches the direction field below?



(a)

$$\begin{aligned} x' &= y \\ y' &= 2y - x \end{aligned}$$

(b)

$$\begin{aligned} x' &= x - 2y \\ y' &= -y \end{aligned}$$

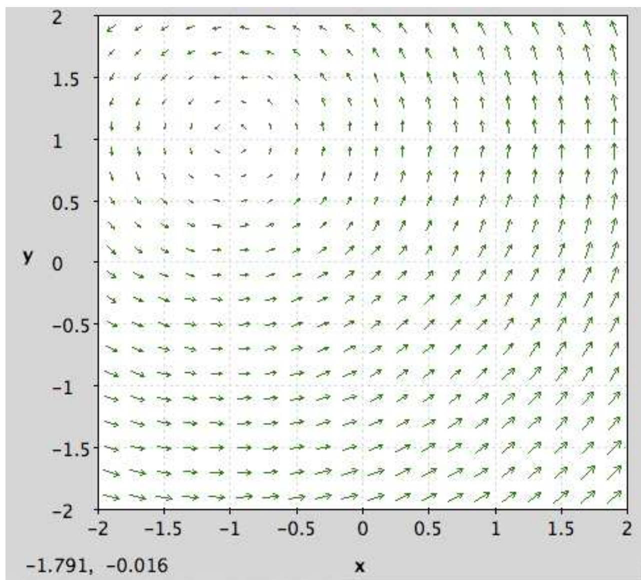
(c)

$$\begin{aligned} x' &= x^2 - 1 \\ y' &= -y \end{aligned}$$

(d)

$$\begin{aligned} x' &= x + 2y \\ y' &= -y \end{aligned}$$

157. Suppose you have the direction field below. At time $t = -2$, you know that $x = -2$ and $y = -.25$. What do you predict is the value of y when $t = 0$?



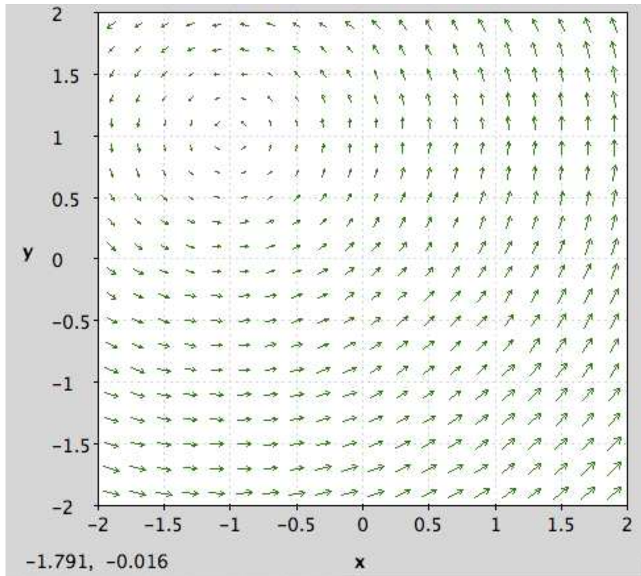
(a) $y \approx -1$

(b) $y \approx 1$

(c) $y \approx 0$

(d) We cannot tell from the information given.

158. Suppose you have the direction field below. We know that at time $t = 0$, we have $x = -2$ and $y = -.25$. The pair $(x(t), y(t))$ is a solution that satisfies the initial conditions. When $y(t) = 0$, about what should $x(t)$ be equal to?



(a) $x \approx -2$

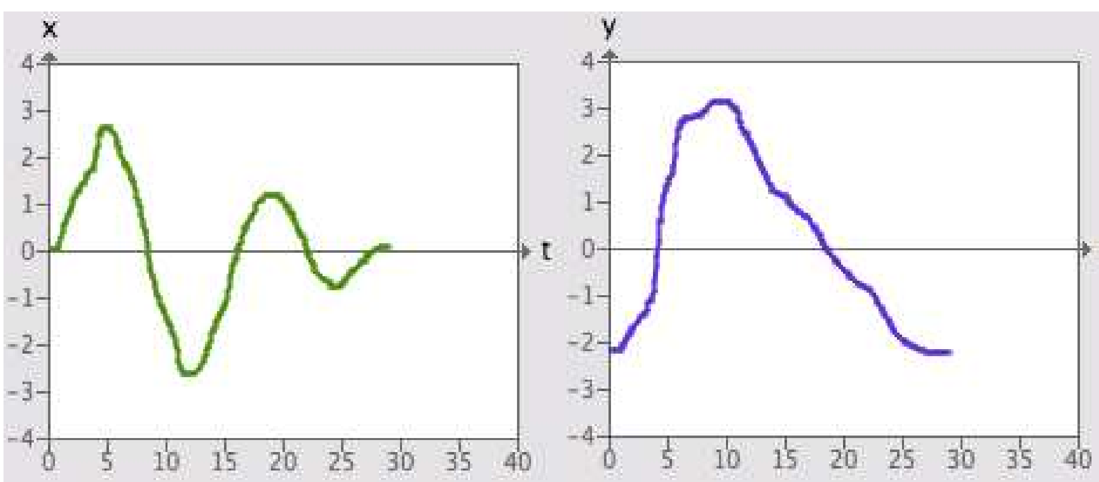
(b) $x \approx -1$

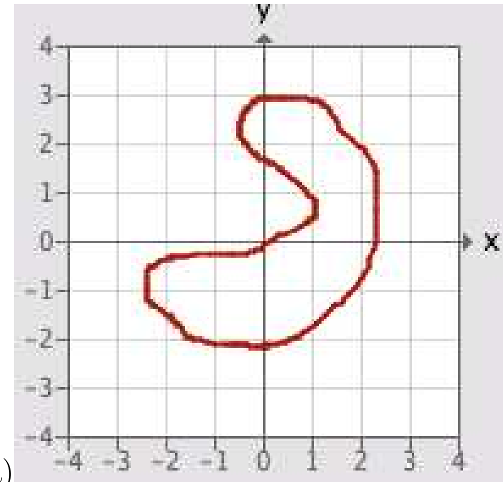
(c) $x \approx 0$

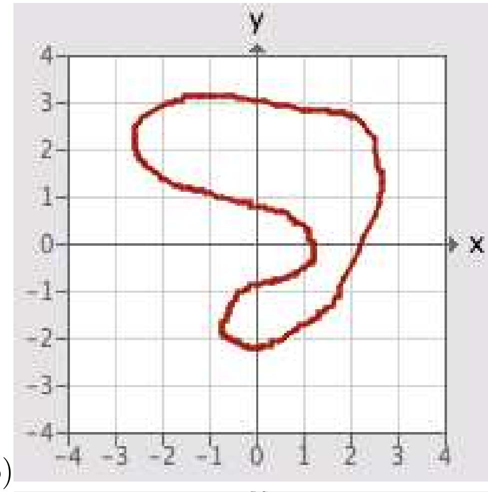
(d) $x \approx 1$

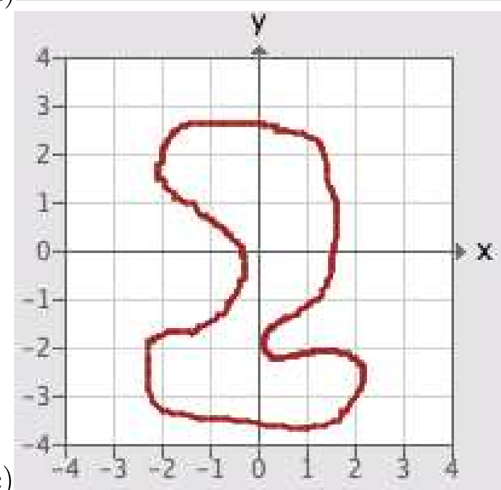
(e) We cannot tell from the information given.

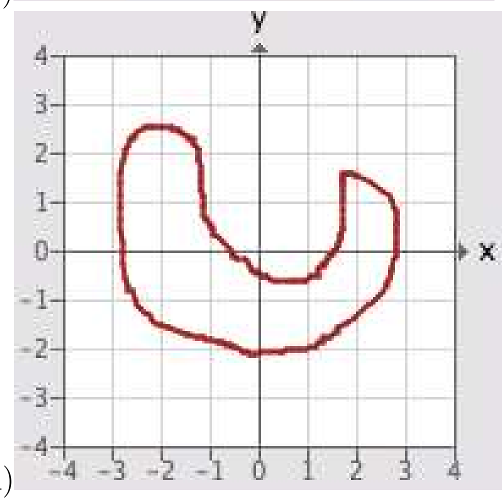
159. Which of the following solution curves in the phase plane might correspond to the solution functions $x(t)$ and $y(t)$ graphed below.



(a) 

(b) 

(c) 

(d) 

Chapter 3: Linear Algebra

Matrix Operations

160. What size is this matrix?

$$\begin{bmatrix} 6 & 11 & -2 \\ 23 & 31 & 5 \end{bmatrix}$$

   (a) 2x3
   (b) 3x2
   (c) 6

161. Let $A = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$
   What is $A + B$?

(a) 71

(b)
$$\begin{bmatrix} 6 & 9 \\ 7 & 11 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 6 & 11 \\ 23 & 31 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 26 & 62 \\ 112 & 268 \end{bmatrix}$$

(e)
$$\begin{bmatrix} 4 & 6 & 2 & 5 \\ 20 & 24 & 3 & 7 \end{bmatrix}$$

162. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$ what is A^T?

(a) $A^T = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$

(b) $A^T = \begin{bmatrix} 2 & 0 & -2 \\ 3 & -1 & 0 \\ 1 & 3 & 4 \end{bmatrix}$

(c) $A^T = \begin{bmatrix} -2 & 0 & 4 \\ 0 & -1 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(d) $A^T = \begin{bmatrix} 1 & 3 & 4 \\ 3 & -1 & 0 \\ 2 & 0 & -2 \end{bmatrix}$

163. If $A = \begin{bmatrix} 4 & 6 \\ 20 & 7 \end{bmatrix}$ what is $5A$?

(a) $5A = \begin{bmatrix} 9 & 6 \\ 20 & 7 \end{bmatrix}$

(b) $5A = \begin{bmatrix} 9 & 11 \\ 25 & 12 \end{bmatrix}$

(c) $5A = \begin{bmatrix} 20 & 6 \\ 20 & 7 \end{bmatrix}$

(d) $5A = \begin{bmatrix} 20 & 30 \\ 100 & 35 \end{bmatrix}$

164. If A is a matrix and c a scalar such that $cA = 0$ (here 0 represents a matrix with all entries equal to zero), then

(a) A is the identity matrix.

(b) $A = 0$

(c) $c = 0$

(d) Both $A = 0$ and $c = 0$

(e) Either $A = 0$ or $c = 0$

(f) We can't deduce anything.

165. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ then calculate the product AB.

(a) $AB = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

(b) $AB = \begin{bmatrix} 10 & 7 \end{bmatrix}$

(c) $AB = \begin{bmatrix} 8 & 4 \\ -3 & -2 \end{bmatrix}$

(d) $AB = \begin{bmatrix} 7 \\ 10 \end{bmatrix}$

(e) None of the above.

(f) This matrix multiplication is impossible.

166. Calculate $\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$.

(a) $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -2 \\ 2 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ -6 & 2 \end{bmatrix}$

(d) None of the above.

(e) This matrix multiplication is impossible.

167. Calculate $\begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$.

(a) $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -2 \\ 2 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ -6 & 2 \end{bmatrix}$

(d) None of the above.

(e) This matrix multiplication is impossible.

168. **True or False** If A and B are square matrices with the same dimensions, then $(A + B) \times (A + B) = A^2 + 2AB + B^2$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

169. If A and B are both 2x3 matrices, then which of the following is not defined?

(a) $A + B$

(b) $A^T B$

(c) BA

(d) AB^T

(e) More than one of the above

(f) All of these are defined.

170. If A is a 2x3 matrix and B is a 3x6 matrix, what size is AB?

(a) 2x6

(b) 6x2

(c) 3x3

(d) 2x3

(e) 3x6

(f) This matrix multiplication is impossible.

171. In order to compute the matrix product AB, what must be true about the sizes of A and B?

(a) A and B must have the same number of rows.

(b) A and B must have the same number of columns.

(c) A must have as many rows as B has columns.

(d) A must have as many columns as B has rows.

172. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{bmatrix}$ what is the (3,2)-entry of AB? (You should be able to determine this without computing the entire matrix product.)

(a) 1

(b) 3

(c) 4

(d) 8

173. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$. where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. If your October sales are 50% more than your May sales, which of the following would represent your October sales?

(a) $M + 50$

(b) $0.5M$

(c) $1.5M$

(d) $M^{.5}$

174. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$. where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. Your June sales are given by the analogous matrix J, where $J = \begin{bmatrix} 6 & 8 \\ 22 & 32 \end{bmatrix}$. Which of the following matrix operations would make sense in this scenario? Be prepared to explain what the result tells you.

(a) $M+J$

(b) $M-J$

(c) $1.2J$

(d) MJ

(e) All of the above make sense.

(f) More than one, but not all, of the above make sense.

175. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$. where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. All tables cost \$350 and all chairs cost \$125, which we represent with the cost vector $C = \begin{bmatrix} 350 \\ 125 \end{bmatrix}$. Which of the following matrix operations could be useful in this scenario? Be prepared to explain what the result tells you.

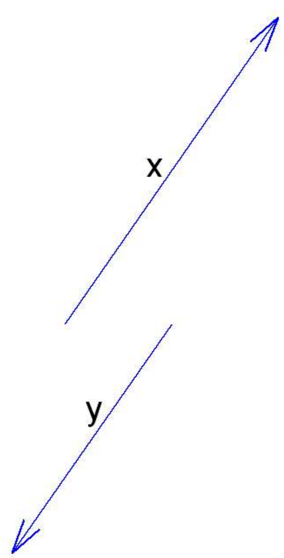
(a) MC

(b) CM

(c) $C^T M$

(d) MC^T

176. **True or False** Given the vectors x and y plotted below and some matrix A, if we know that $Ax = 0$, this means that $Ay = 0$ as well.



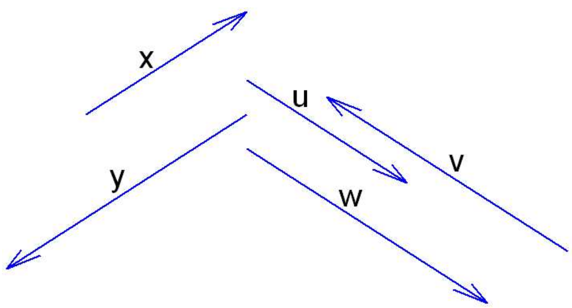
(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

177. Given the vectors x, y, u, v, and w plotted below and some matrix A, if we know that $Ax = u$, what does this tell us about the product Ay?



(a) $Ay = u$

(b) $Ay = v$

(c) $Ay = w$

(d) We cannot say anything about Ay without knowing more about A.

178. Let A, B, C be 3 matrices such that the product ABC is defined. What is $(ABC)^T$?

(a) $(ABC)^T = A^T B^T C^T$

(b) $(ABC)^T = B^T C^T A^T$

(c) $(ABC)^T = C^T A^T B^T$

(d) $(ABC)^T = C^T B^T A^T$

Dot Products

179. What is the dot product of $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$?

(a) $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$

(b) 5

(c) 0

(d) The dot product cannot be computed for these vectors.

180. What is the dot product of $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$?

(a) $\begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$

(b) -4

(c) 0

(d) The dot product cannot be computed for these vectors.

181. The magnitude of a vector v is defined to be its dot product with itself $v \cdot v$. What is the magnitude of the vector $(2, -1, -1)$?

(a) 0

(b) 2

(c) 4

(d) 6

182. It is possible for a vector to have a negative magnitude?

(a) Yes

(b) No

(c) Not enough information is given

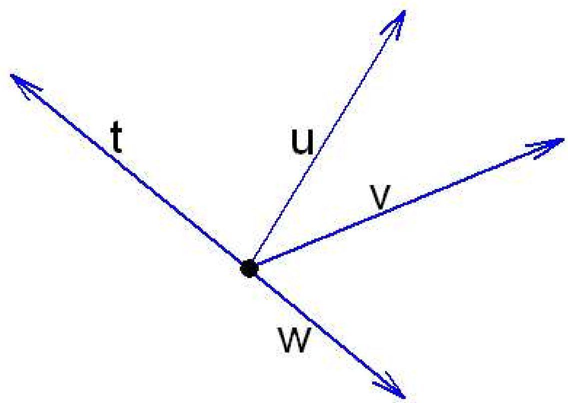
183. What can we say about two vectors whose dot product is negative?

(a) The vectors are orthogonal.

(b) The angle between the two vectors is less than $90°$.

(c) The angle between the two vectors is greater than $90°$.

184. Rank the dot products $u \cdot v$, $u \cdot t$ and $u \cdot w$.



(a) $u \cdot v > u \cdot w > u \cdot t$
(b) $u \cdot v > u \cdot t > u \cdot w$
(c) $u \cdot w > u \cdot t > u \cdot v$
(d) $u \cdot w > u \cdot v > u \cdot t$

185. **True or False** If x and y are $n \times 1$ vectors, then $x^T y = y^T x$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

186. **True or False** If x and y are $n \times 1$ vectors, then xy^T is an $n \times n$ matrix.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

187. **True or False** If x and y are $n \times 1$ vectors, then $xy^T = yx^T$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

188. **True or False** If x and y are $n \times 1$ nonzero vectors, then xy^T is an $n \times n$ matrix with rank 1.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

189. If A and B are matrices which can be multiplied, then the (i, j)-entry of AB is

(a) $\left[\text{the } i^{\text{th}} \text{ row of } A\right] \cdot \left[\text{the } j^{\text{th}} \text{ column of } B\right]$
(b) $\left[\text{the } i^{\text{th}} \text{ column of } A\right] \cdot \left[\text{the } j^{\text{th}} \text{ row of } B\right]$
(c) None of the above

190. When we are in the vector space of real valued functions, it is often useful to have the equivalent of a dot product, which we call an inner product: We define the inner product of two functions $f(x)$ and $g(x)$ as $\langle f, g\rangle \equiv \int_a^b f(x)g(x)dx$. Consider the functions $f(x) = \sin 2\pi x$ on the interval $(a, b) = (0, 1)$. What is the inner product of this function with itself $\langle f, f\rangle$?

(a) 0
(b) $\frac{1}{2}$
(c) 1
(d) 2
(e) This is not a meaningful statement.

191. $\langle \sin(2\pi x), \sin(4\pi x)\rangle \equiv \int_0^1 \sin(2\pi x)\sin(4\pi x)dx = 0$. What does this mean?

(a) These are parallel functions.
(b) These are orthogonal functions.
(c) These are acute functions.
(d) These are obtuse functions.

Orthogonal Sets

192. Which of the following sets of vectors is *not* an orthogonal set?

(a) $(1, 1, 1), (1, 0, -1)$

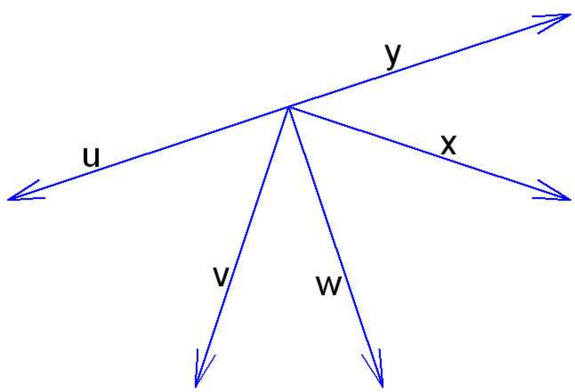
(b) $(2, 3), (-6, 4)$

(c) $(3, 0, 0, 2), (0, 1, 0, 1)$

(d) $(0, 2, 0), (-1, 0, 3)$

(e) $(\cos\theta, \sin\theta), (\sin\theta, -\cos\theta)$

193. Which of the following sets of vectors is *not* an orthogonal set?



(a) u, w

(b) x, v

(c) v, y

(d) u, w, y

(e) More than one of the above

(f) None of the above

194. Let A be a square matrix whose columns are mutually orthogonal, nonzero vectors. Which of the following are true?

(a) The dot product of any two different column vectors is zero.

(b) The set of column vectors is linearly independent.

(c) $\det(A) \neq 0$.

(d) For any b, there is a unique solution to $Ax = b$.

(e) All of the above.

195. **True or False** If two vectors are linearly independent, they must be orthogonal.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

196. **True or False** Any orthogonal set of nonzero vectors that spans a vector space must be a basis for that space.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

197. Let A be any matrix. Which of the following are true?

(a) The row space of A and the nullspace of A are orthogonal to each other.
(b) The column space of A and the row space of A are orthogonal to each other.
(c) The column space of A and the nullspace of A are orthogonal to each other.
(d) Exactly two of (a), (b), and (c) are true.
(e) All of (a), (b), and (c) are true.

198. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the row space of A?

(a) $(1, 1, -1)$
(b) (1, 4, 2)
(c) (0, 0, 5)
(d) $(-1, 0, 1)$

199. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the column space of A?

(a) $(1, 1, -1)$
(b) $(1,\ 4,\ 2)$
(c) $(0, 1, -2)$
(d) $(2, 0, 2)$

200. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the nullspace of A?

(a) $(1, 1, -1)$
(b) $(1,\ 4,\ 2)$
(c) $(0, 1, -2)$
(d) $(2, 0, 2)$

201. Which of the following sets of vectors is an orthonormal set?

(a) $(1, 1, 1), (1, 0, -1)$
(b) $(2, 3), (-6, 4)$
(c) $(0, 2, 0), (-1, 0, 3)$
(d) $(\cos\theta, \sin\theta), (\sin\theta, -\cos\theta)$

202. Let A be a matrix whose columns are mutually orthogonal. Which of the following must be true? Try several examples of matrices with mutually orthogonal columns to build your intuition, then try to provide a proof.

(a) A is symmetric.
(b) $A^{-1} = A^T$.
(c) $A^T A$ is diagonal.
(d) $\det(A) \neq 0$.
(e) All of the above must be true.
(f) More than one, but not all, of the above must be true.

203. Let M be any matrix. **True or False** The columns of M are orthonormal if and only if $M^T M$ is an identity matrix.

(a) True, and I am very confident

(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

204. Let Q be a square matrix with orthonormal columns. **True or False** $Q^{-1} = Q^T$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

205. **True or False** Any set of nonzero orthogonal vectors must also be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

206. **True or False** The only orthonormal basis for $\Re^2$ is $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

Systems of Equations

207. What is the solution to the following system of equations?

$$\begin{aligned} 2x + y &= 3 \\ 3x - y &= 7 \end{aligned}$$

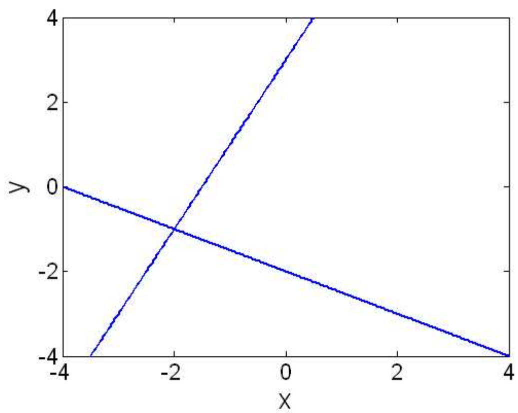
(a) $x = 4$ and $y = -5$
(b) $x = 4$ and $y = 5$
(c) $x = 2$ and $y = -1$

(d) $x = 2$ and $y = 1/2$

(e) There are an infinite number of solutions to this system.

(f) There are no solutions to this system.

208. Which of the following systems of equations could be represented in the graph below?



(a) $3x + 3y = -6$, $x + 2y = 3$

(b) $x - y = -5$, $2x + y = 4$

(c) $-8x + 4y = 12$, $2x + 4y = -8$

(d) $-x + 3y = 9$, $2x - y = 4$

209. What is the solution to the following system of equations?

$$\begin{aligned} 2x + y &= 3 \\ 4x + 2y &= 6 \end{aligned}$$

(a) $x = 0$ and $y = 0$

(b) $x = 2$ and $y = -1$

(c) $x = 0$ and $y = 1$

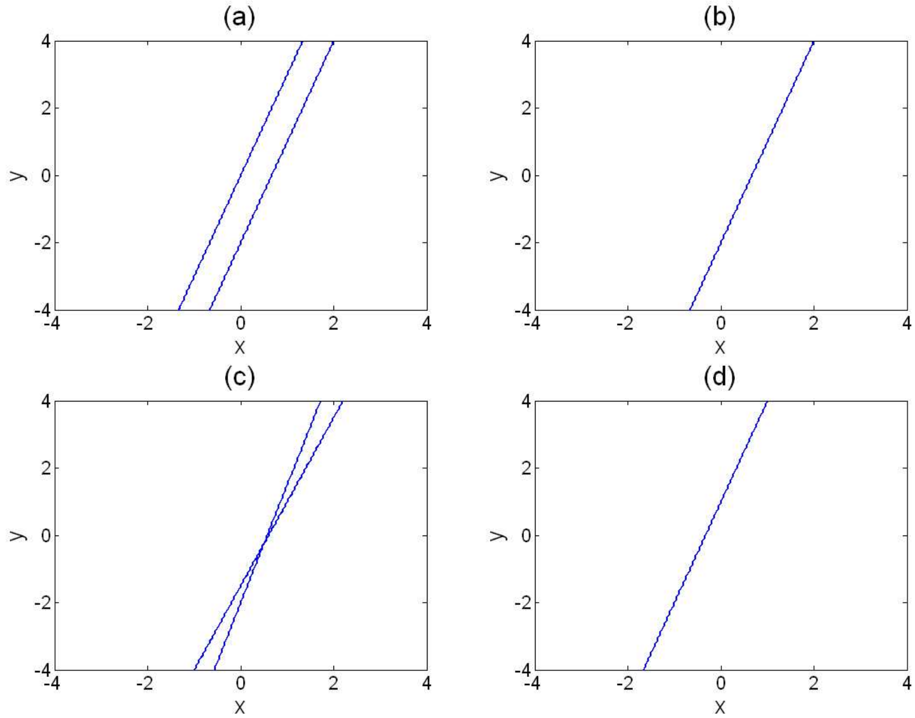
(d) $x = 0$ and $y = 3$

(e) There are an infinite number of solutions to this system.

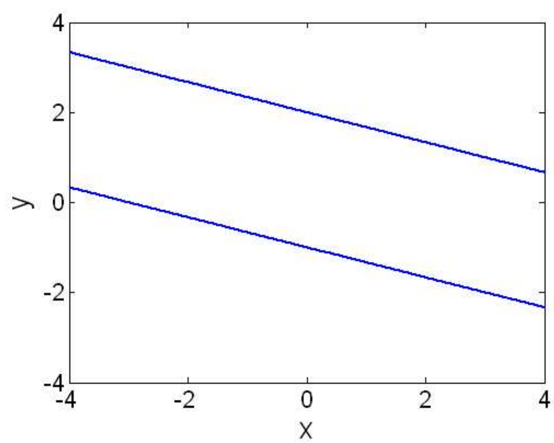
(f) There are no solutions to this system.

210. Which of the graphs below could represent the following linear system?

$$\begin{aligned} 3x - y &= 2 \\ -9x + 3y &= -6 \end{aligned}$$



211. Which of the following systems of equations could be represented in the graph below?



(a) $-x + 3y = 6,\ 2x + 6y = -6$

(b) $-x + 3y = 6,\ 2x + 6y = 12$

(c) $x + 3y = 6,\ 2x + 6y = 12$

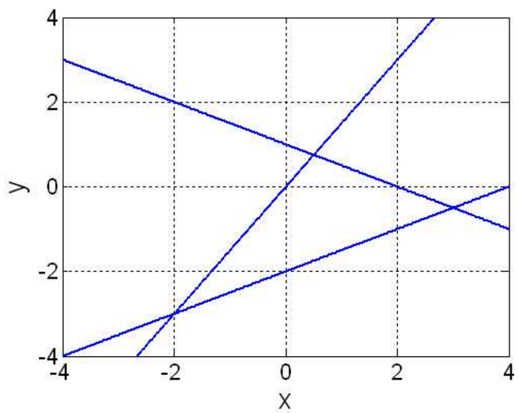
(d) $x + 3y = 6,\ x + 3y = -3$

212. What is the solution to the following system of equations?

$$\begin{aligned} -3x + 2y &= 4 \\ 12x - 8y &= 10 \end{aligned}$$

(a) $x = -4/3$ and $y = 0$
(b) $x = 1/2$ and $y = -1/2$
(c) $x = 0$ and $y = 2$
(d) $x = 1/3$ and $y = 5/2$
(e) There are an infinite number of solutions to this system.
(f) There are no solutions to this system.

213. We have a system of three linear equations with two unknowns, as plotted in the graph below. How many solutions does this system have?



(a) 0
(b) 1
(c) 2
(d) 3
(e) Infinite

214. A system of linear equations could *not* have exactly ________ solutions.

(a) 0
(b) 1
(c) 2
(d) infinite
(e) All of these are possible numbers of solutions to a system of linear equations.

215. The system

$$\begin{aligned} x + y &= 2 \\ 2x + 2y &= 4 \end{aligned}$$

has an infinite number of solutions. Which of the following describes the set of solutions to this system?

(a) $x = 1$ and $y = 1$

(b) $x = 2 - t$ and $y = t$

(c) x and y could each be anything.

(d) None of the above

216. Which of the following options describes the set of solutions to the system below?

$$\begin{aligned} x + y &= 1 \\ x - y &= 0 \\ 2x + y &= 3 \end{aligned}$$

(a) $x = 1 - t$ and $y = t$

(b) $x = 1$ and $y = 1$

(c) No solution exists

(d) None of the above

217. Which of the following options describes the set of solutions to the system below?

$$\begin{aligned} x + y &= 2 \\ 2x - y &= -2 \\ x - 2y &= -4 \end{aligned}$$

(a) $x = t$ and $y = 2 - t$

(b) $x = 0$ and $y = 2$

(c) no solution exists

(d) None of the above

218. $x = 3 - 2t$ and $y = t$ represent the set of solutions to a system of equations. What line in $\Re^2$ does this set of solutions represent?

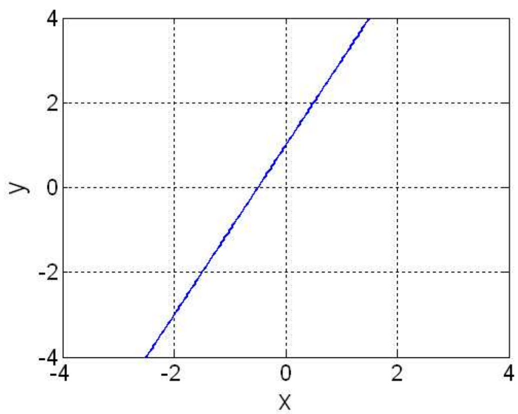
(a) $x + 2y = 3$

(b) $x - 2y = 3$

(c) $x + y = 3 - t$

(d) It is impossible to answer this question with the information given.

219. The set of solutions to a system of linear equations is plotted below. Which of the following parameterizations represents this solution set?



(a) $x = 2t$ and $y = 4t + 1$

(b) $x = \frac{1}{2}t - \frac{1}{2}$ and $y = t$

(c) $x = t - 1$ and $y = 2t - 1$

(d) $x = t$ and $y = 2t + 1$

(e) All of the above

220. A certain mini-golf course does not list their prices. I paid \$26.25 for 3 children and 4 adults. The group in front of me had paid \$25.50 for 6 children and 2 adults. Which system of equations would allow us to determine the prices for children and adults?

(a)

$$\begin{aligned} 3x + 6y &= 26.25 \\ 4x + 2y &= 25.50 \end{aligned}$$

(b)

$$\begin{aligned} 3x + 4y &= 26.25 \\ 6x + 2y &= 25.50 \end{aligned}$$

(c)

$$\begin{aligned} 26.25x + 25.50y &= 51.75 \\ 9x + 6y &= 15 \end{aligned}$$

(d)

$$\begin{aligned}(26.25/3)x + (26.25/4)y &= 0\\(25.50/6)x + (25.50/6)y &= 0\end{aligned}$$

221. A system of 3 linear equations with 3 variables could not have exactly ____________ solutions.

    (a) 0
    (b) 1
    (c) 2
    (d) 3
    (e) More than one of (a)-(d) are impossible.
    (f) All of (a)-(d) are possible numbers of solutions.

222. A linear equation with two variables can be geometrically represented as a line in $\Re^2$. How can we best represent a linear equation with three variables?

    (a) As a line in $\Re^2$
    (b) As a line in $\Re^3$
    (c) As a plane in $\Re^3$
    (d) As a volume in $\Re^3$

223. We find that a system of three linear equations in three variables has an infinite number of solutions. How could this happen?

    (a) We have three equations for the same plane.
    (b) At least two of the equations must represent the same plane.
    (c) The three planes intersect along a line.
    (d) The planes represented are parallel.
    (e) More than one of the above are possible.

224. We consider a system of three linear equations in three variables, and visualize the graph of each equation as a plane in $\Re^3$. Suppose no solutions exist to this system. This means that

    (a) all three planes must be parallel.

(b) at least two of the planes must be parallel.
(c) at least two of the equations represent the same plane.
(d) none of these planes ever intersects with another.
(e) None of the above

225. We have a system of four linear equations in four variables. We can think about the graph of each equation as a 3-dimensional volume in $\Re^4$. Which of the following could geometrically represent the solutions to this system?

(a) A point in $\Re^4$
(b) A line in $\Re^4$
(c) A plane in $\Re^4$
(d) A three dimensional volume in $\Re^4$
(e) All of the above
(f) None of the above

226. How can we geometrically represent the parametric equations $x = 2t$, $y = -t + 1$, and $z = t$?

(a) A line in $\Re^2$
(b) A line in $\Re^3$
(c) A plane in $\Re^3$
(d) A volume in $\Re^3$

227. A system of 5 linear equations and 7 variables could not have exactly ____________ solutions.

(a) 0
(b) 1
(c) infinite
(d) More than one of these is impossible.
(e) All of these are possible numbers of solutions.

228. A system of 8 linear equations and 6 variables could not have exactly ____________ solutions.

(a) 0

(b) 1

(c) infinite

(d) More than one of these is impossible.

(e) All of these are possible numbers of solutions.

229. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

whole wheat flour	soy flour	
0.5	0.5	Standard Blend
0.8	0.2	Extra Wheat
0.3	0.7	Extra Soy

A customer comes in who wants one pound of a blend that is 60% wheat and 40% soy. Which system of equations below would allow us to solve for the amount of each blend needed to fulfill this special request?

(a)

$$\begin{aligned} 0.5x_1 + 0.5x_2 &= 1 \\ 0.8x_1 + 0.2x_2 &= 1 \\ 0.3x_1 + 0.7x_2 &= 1 \end{aligned}$$

(b)

$$\begin{aligned} 0.5x_1 + 0.5x_2 &= 0.6 \\ 0.8x_1 + 0.2x_2 &= 0.4 \\ 0.3x_1 + 0.7x_2 &= 0 \end{aligned}$$

(c)

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 1 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 1 \end{aligned}$$

(d)

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 0.6 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 0.4 \end{aligned}$$

230. In the previous question you set up a system of equations so that you could find the amount of each blend needed to make a new mixture. How many solutions must this system have? (You do not need to solve the system.)

(a) 0
(b) 1
(c) 2
(d) 3
(e) Infinite

231. The previous two questions dealt with the system

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 0.6 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 0.4 \end{aligned}$$

In the context given, what quantity or unit does 0.6 represent?

(a) pounds
(b) %
(c) pounds^2
(d) pounds per %
(e) 0.6 does not have units

Matrix Representations of Systems of Equations

232. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your current inventory is 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs. Which matrix would best represent this information?

(a)
$$\begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 4 & 6 \\ 24 & 20 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 6 & 4 \\ 20 & 24 \end{bmatrix}$$

(d) They all represent the information equally well.

233. Which augmented matrix represents the following system of equations?

$$\begin{aligned} x + 2y &= 3 \\ 4y + 5x &= 6 \end{aligned}$$

(a)

$$\begin{bmatrix} 0 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

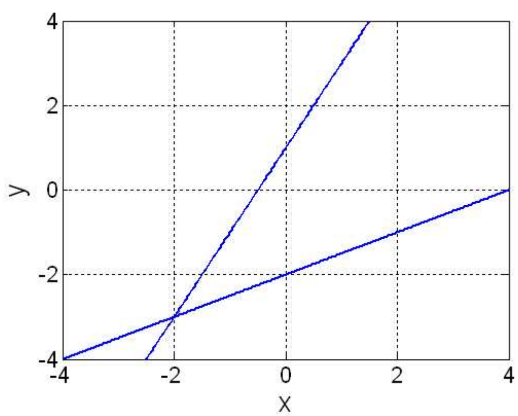
(c)

$$\begin{bmatrix} 1 & 2 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0 & 2 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

234. The rows of which augmented matrix represent equations plotted below?



(a)

$$\begin{bmatrix} 1 & 0 & 3 \\ -3 & -3 & 4 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2 & 2 & -3 \\ 0 & 1 & 4 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 2 & -4 \\ 3 & -2 & -2 \end{bmatrix}$$

(d)

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$$

235. Which matrix represents the following system of equations?

$$\begin{aligned} x &= 6 \\ y &= 3 \end{aligned}$$

(a)

$$\begin{bmatrix} 1 & 6 \\ 1 & 3 \end{bmatrix}$$

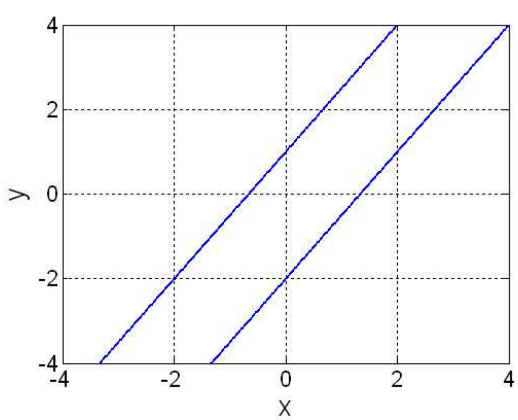
(b)

$$\begin{bmatrix} 1 & 1 & 9 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 3 \end{bmatrix}$$

236. The rows of which augmented matrix represent the equations plotted below?



(a)

$$\begin{bmatrix} 3 & -2 & 4 \\ 6 & -4 & 8 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 3 & -2 & -2 \\ 3 & -2 & 4 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 3 & 2 & -2 \\ 3 & -2 & 2 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 6 & -4 & 8 \\ 3 & 2 & -2 \end{bmatrix}$$

237. What is the solution to the system of equations represented with this augmented matrix?
$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

(a) $x = 2$, $y = 3$, $z = 4$

(b) $x = -1$, $y = 1$, $z = 1$

(c) There are an infinite number of solutions.

(d) There is no solution.

(e) We can't tell without having the system of equations.

238. What is the solution to the system of equations represented with this augmented matrix?
$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

(a) $x = 2$, $y = 3$, $z = 4$

(b) $x = -1$, $y = 1$, $z = 1$

(c) There are an infinite number of solutions.

(d) There is no solution.

(e) We can't tell without having the system of equations.

239. Suppose we want to graph the equations represented by the rows of the augmented matrix below. What will they look like?

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

(a) These equations represent two lines that intersect at $x = 2$ and $y = 3$.

(b) These equations represent three parallel planes.

(c) These equations represent three planes that are not parallel, but which do not share a common point of intersection.

(d) These equations cannot be represented geometrically.

240. What is the solution to the system of equations represented with this augmented matrix?

$$\begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) $x = 2$, $y = 3$, $z = 4$

(b) $x = -1$, $y = 1$, $z = 1$

(c) There are an infinite number of solutions.

(d) There is no solution.

(e) We can't tell without having the system of equations.

241. Suppose we want to graph the equations represented by the rows of the augmented matrix below. What will they look like?

$$\begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) These equations represent two equations for the same plane.

(b) These equations represent three equations for the same plane.

(c) These equations represent two planes that have a line of points in common.

(d) The intersection of these linear equations is represented by a plane in $\Re^3$.

(e) These equations cannot be represented geometrically.

Gaussian Elimination

242. Which of the following operations on an augmented matrix could change the solution set of a system?

(a) Interchanging two rows

(b) Multiplying one row by any constant

(c) Adding one row to an other

(d) Adding a multiple of one row to an other

(e) None of the above

(f) More than one of the above (which ones?)

243. Which of the following matrices is NOT row equivalent to the one below? In other words, which matrix could you NOT get from the matrix below through elementary row operations?

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & 1 & 3 & 4 \\ 1 & 2 & 0 & 4 \end{bmatrix}$$

(a)

$$\begin{bmatrix} 2 & 4 & 0 & 8 \\ 0 & 1 & 3 & 4 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & 1 & 3 & 4 \\ 1 & 3 & 3 & 8 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 0 & 3 & 4 \\ 2 & 1 & 0 & 4 \end{bmatrix}$$

(d) More than one of the above

(e) All are possible through elementary row operations.

244. Which of the following matrices is row equivalent to the one below? In other words, which matrix could you get from the matrix below through elementary row operations?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a)

$$\begin{bmatrix} 2 & 5 & 7 \\ 0 & 1 & 3 \\ 4 & 8 & 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 3 & 1 & 5 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} -3 & 1 & 3 \\ -2 & 1 & 0 \\ 3 & 9 & 2 \end{bmatrix}$$

(d) More than one of the above

(e) All are possible through elementary row operations.

245. Which of the following matrices is NOT row equivalent to the one below? In other words, which matrix could you NOT get from the matrix below through elementary row operations?

$$\begin{bmatrix} 6 & 0 & 4 & 7 \\ 2 & 0 & 1 & 9 \\ 5 & 0 & 3 & 5 \end{bmatrix}$$

(a)

$$\begin{bmatrix} 12 & 0 & 8 & 14 \\ 2 & 0 & 1 & 9 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

(b)

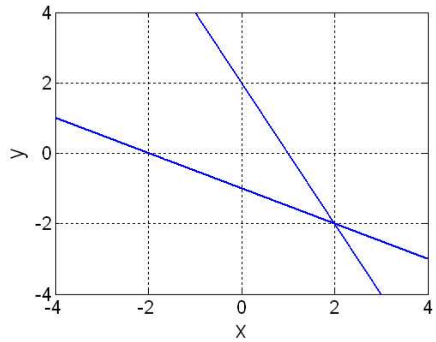
$$\begin{bmatrix} 12 & 0 & 8 & 14 \\ 0 & 0 & 1 & -20 \\ 2 & 1 & 3 & 0 \end{bmatrix}$$

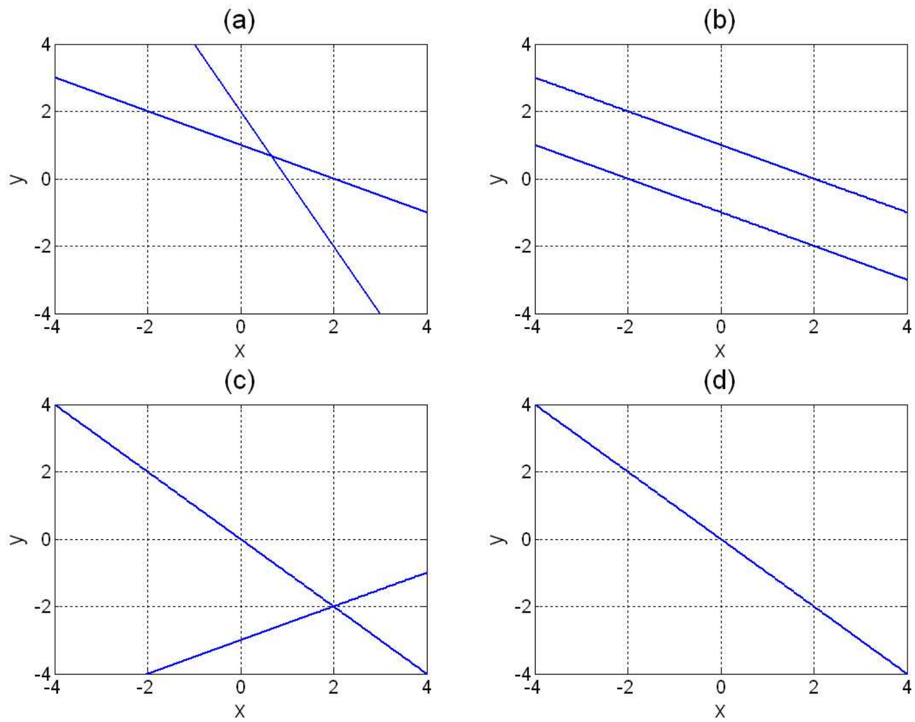
(c)

$$\begin{bmatrix} 6 & 0 & 4 & 7 \\ 2 & 0 & 1 & 9 \\ 7 & 0 & 4 & 14 \end{bmatrix}$$

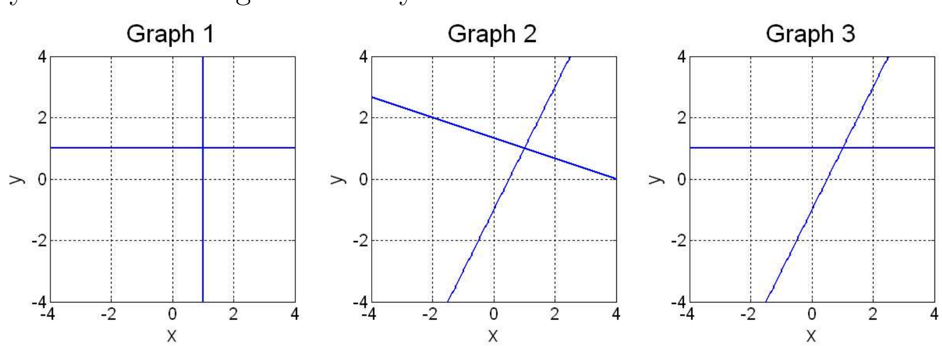
(d) All are possible through elementary row operations.

246. A linear system of equations is plotted below. We create an augmented matrix to represent this linear system, then perform a series of elementary row operations. Which of the following graphs could represent the result of these row operations?





247. We have a system of two linear equations and two unknowns which we solve by performing Gaussian elimination on an augmented matrix. Along the way we create the graphs below, showing geometrical representations of the initial system, the system at an intermediate step in the row reduction process, and the system after it has been put into reduced row echelon form. Put these graphs in order, starting with the initial system and ending with the system in reduced row echelon form.



(a) Graph 2, Graph 3, Graph 1
(b) Graph 1, Graph 3, Graph 2
(c) Graph 1, Graph 2, Graph 3
(d) Graph 2, Graph 1, Graph 3
(e) Graph 3, Graph 2, Graph 1

248. What is the value of a so that the linear system represented by the following matrix would have infinitely many solutions?

$$\begin{bmatrix} 2 & 6 & 8 \\ 1 & a & 4 \end{bmatrix}$$

(a) $a = 0$
(b) $a = 2$
(c) $a = 3$
(d) $a = 4$
(e) This is not possible.
(f) More than one of the above

249. We start with a system of two linear equations in two variables and we translate this system into an augmented matrix M. After performing Gaussian elimination, putting this matrix into reduced row echelon form, we get the matrix R which tells us that this system has no solution. How could we geometrically represent the linear equations contained in the rows of the augmented matrix R?

(a) We can represent the equations of R as two parallel lines.
(b) We can represent the equations of R as two lines that may not be parallel.
(c) We can represent the equations of R as a single line.
(d) The equations of R cannot be represented geometrically.

250. We start with a system of three linear equations in three variables and we translate this system into an augmented matrix M. After performing Gaussian elimination, putting this matrix into reduced row echelon form, we get the matrix R which tells us that this system has no solution. How could we best geometrically represent the linear equations contained in the rows of the augmented matrix M?

(a) We can represent the equations of M as three parallel lines.
(b) We can represent the equations of M as three parallel planes.

(c) We can represent the equations of M as three planes, where at least two must be parallel.

(d) We can represent the equations of M as three planes, where none of the planes ever intersects with another.

(e) We can represent the equations of M as three planes, which do not share any points in common.

(f) The equations of M cannot be represented geometrically.

251. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

whole wheat flour	soy flour	
0.5	0.5	Standard Blend
0.8	0.2	Extra Wheat
0.3	0.7	Extra Soy

A customer comes in who wants one pound of a blend that is 60% wheat and 40% soy. We can solve the following system of equations to determine the amount of Standard Blend (x_1), Extra Wheat Blend (x_2), and Extra Soy Blend (x_3) needed to create this special mixture.

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 0.6 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 0.4 \end{aligned}$$

If we form an augmented matrix for this system, the reduced row echelon form is $R = \left[\begin{array}{cccc} 1 & 0 & 5/3 & 2/3 \\ 0 & 1 & -2/3 & 1/3 \end{array} \right]$.

If the store is out of Extra Soy Blend, how much of each of the other blends is needed?

(a) 2/3 pound of Standard Blend and 1/3 pound of Extra Wheat Blend

(b) 5/3 pound of Standard Blend and 2/3 pound of Extra Wheat Blend

(c) There are an infinite number of options for the amounts of Standard and Extra Wheat Blend.

(d) It is not possible to create this mixture without Extra Soy Blend.

252. Referring to the previous question, if the store is out of Extra Wheat Blend (x_2), how much of each of the other blends is needed to make the special mixture?

(a) 2/3 pound of Standard Blend and 1/3 pound of Extra Soy Blend

(b) 1/6 pound of Standard Blend and 1/2 pound of Extra Soy Blend

(c) There are an infinite number of options for the amounts of Standard Blend and Extra Wheat Blend.

(d) It is not possible to create this mixture without Extra Wheat Blend.

253. Referring to the previous two questions, what values are realistic for x_3 in this context?

(a) x_3 can be any value.

(b) $x_3 \geq 0$

(c) $\frac{2}{3} \leq x_3 \leq \frac{5}{3}$

(d) $-\frac{1}{2} \leq x_3 \leq \frac{2}{5}$

(e) $0 \leq x_3 \leq \frac{2}{5}$

254. Let R be the reduced row echelon form of a $n \times n$ matrix A. Then

(a) R is the identity.

(b) R has at least one row of zeros.

(c) None of the above.

(d) All of the above are possible but there exist also other possibilities.

(e) The two possibilities above are the only ones.

(f) We can't tell without having the matrix A.

Solution Sets of Linear Systems

255. Which of the following are solutions to the system of equations?

$$\begin{aligned} 2x + y + 2z &= 0 \\ -x + 2y - 6z &= 0 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 6 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ -2 \end{bmatrix}$

(e) None of the above.

(f) More than one of the above.

256. What is the solution to the following system of equations?

$$\begin{aligned} x + 2y + z &= 0 \\ x + 3y - 2z &= 0 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 1 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 0 \end{bmatrix}$

(e) None of the above.

(f) More than one of the above.

257. What is the solution to the following system of equations?

$$\begin{aligned} x + 2y + z &= 3 \\ x + 3y - 2z &= 4 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} s$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(d) None of the above.

(e) More than one of the above.

258. What is the solution to the following system of equations?

$$\begin{aligned} x + 2y + z &= -2 \\ x + 3y - 2z &= 1 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

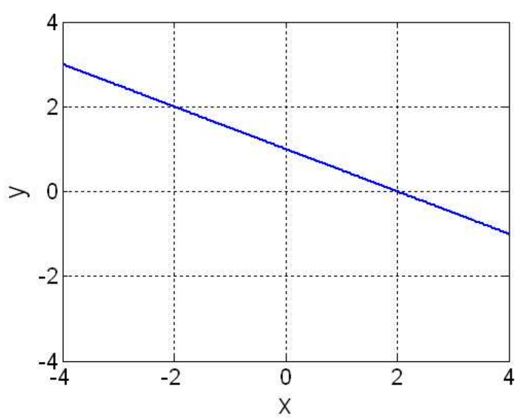
(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 7 \\ -3 \\ -1 \end{bmatrix} s$

(d) None of the above.

(e) More than one of the above.

259. The set of solutions to a system of linear equations is plotted below. Which of the following expressions represents this solution set?



(a) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1/2 \end{bmatrix} s$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1/2 \\ 2 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1/2 \\ 2 \end{bmatrix} s$

(d) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} s$

(e) None of the above.

(f) More than one of the above.

260. The set of solutions to a linear system are represented by the expression below. How can we geometrically represent this solution set?

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} t$$

(a) As a line in $\Re^2$

(b) As a line in $\Re^3$

(c) As a plane in $\Re^3$

(d) As a volume in $\Re^3$

(e) None of the above

261. Let $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. If R is the reduced row echelon form of the augmented matrix for the system $Ax = b$, what are the solutions to that system?

(a) $x_1 = 1, x_2 = 1,$ and $x_3 = 2$

(b) $x_1 = 1, x_2 = 1, x_3 = 2,$ and $x_4 = 0$

(c) $x_1 = -t, x_2 = -t, x_3 = -2t,$ and $x_4 = t$

(d) There are no solutions to this system.

262. Let $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. If R is the reduced row echelon form of the coefficient matrix for the system $Ax = 0$, what are the solutions to that system?

(a) $x_1 = 1, x_2 = 1,$ and $x_3 = 2$

(b) $x_1 = 1, x_2 = 1, x_3 = 2,$ and $x_4 = 0$

(c) $x_1 = -t, x_2 = -t, x_3 = -2t,$ and $x_4 = t$

(d) There are no solutions to this system.

263. Let matrix R be the reduced row echelon form of matrix A. **True or False** The solutions to $Rx = 0$ are the same as the solutions to $Ax = 0$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

264. Let matrix R be the reduced row echelon form of matrix A. **True or False** The solutions to $Rx = b$ are the same as the solutions to $Ax = b$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

265. Consider a homogeneous linear system with n unknowns. Suppose the reduced row echelon form of its augmented matrix has $r \leq n$ nonzero rows. We can conclude that:

(a) $x_1 = 0, x_2 = 0, \ldots, x_n = 0$ is a solution to the system.
(b) The system has $n - r$ free variables (parameters).
(c) The system has infinitely many solutions.
(d) None of the above.
(e) More than one of the above.

Dimension and Rank

266. Let $A = \begin{bmatrix} 5 & 4 & -8 & 1 \\ 1 & 3 & 4 & 8 \\ 0 & 2 & 1 & 3 \\ -1 & -2 & 4 & 1 \end{bmatrix}$. The reduced row echelon form of A is $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. What is the rank of A?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

267. Suppose a 4×4 matrix A has rank 3. Are the columns of A linearly independent?

(a) Yes, they are linearly independent.
(b) No, they are not linearly independent.
(c) We do not have enough information to decide.

268. Suppose a 4×4 matrix A has rank 4. How many solutions does the system $Ax = b$ have?

(a) 0
(b) 1
(c) Infinite
(d) Not enough information is given.

269. Suppose a 4×4 matrix A has rank 3. How many solutions does the system $Ax = b$ have?

(a) 0
(b) 1
(c) Infinite
(d) Not enough information is given.

270. Suppose a 4×4 matrix A has rank 3. If it is known that (4, 5, 0, 1) is a solution to the system $Ax = b$, then how many solutions does $Ax = b$ have?

(a) 1
(b) Infinite
(c) Not enough information is given.

271. Suppose a 5×5 matrix A has rank 3. If it is known that (-1, 4, 2, 0, 3) is a solution to the system $Ax = b$, then how many parameters does the solution set have?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

(f) Not enough information is given.

272. **True or False** If $AX = BX$ for all matrices X where the products are defined, then A and B have to be the same matrix.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

273. **True or False** If $Ax = Bx$ for all vectors x where the products are defined, then A and B have to be the same matrix.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

Matrix Inverses

274. Which of the following matrices does not have an inverse?

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
(b) $\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}$
(c) $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$
(d) $\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$
(e) More than one of the above do not have inverses.
(f) All have inverses.

275. When we put a matrix A into reduced row echelon form, we get the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$. This means that

(a) Matrix A has no inverse.
(b) The matrix we have found is the inverse of matrix A.
(c) Matrix A has an inverse, but this isn't it.
(d) This tells us nothing about whether A has an inverse.

276. Let $A = \begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$. What is A^{-1}?

(a) $\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$.
(b) $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$.
(c) $\begin{bmatrix} 0 & 1/4 \\ 1/2 & 0 \end{bmatrix}$.
(d) $\begin{bmatrix} 0 & 1/2 \\ 1/4 & 0 \end{bmatrix}$.

277. We find that for a square coefficient matrix A, the homogeneous matrix equation $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, has only the trivial solution $X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. This means that

(a) Matrix A has no inverse.
(b) Matrix A has an inverse.
(c) This tells us nothing about whether A has an inverse.

278. **True or False** If A, B, and C are square matrices and we know that $AB = AC$, this means that matrix B is equal to matrix C.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

279. **True or False** Suppose that A, B, and C are square matrices, and $CA = B$, and A is invertible. This means that $C = A^{-1}B$.

(a) True, and I am very confident

(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

280. We know that $(5A)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. What is matrix A?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
(c) $\begin{bmatrix} 1/5 & 0 \\ 0 & 1/5 \end{bmatrix}$
(d) $\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$
(e) There is no matrix A which solves this equation.

281. A and B are invertible matrices. If $AB = C$, then what is the inverse of C?

(a) $C^{-1} = A^{-1}B^{-1}$
(b) $C^{-1} = B^{-1}A^{-1}$
(c) $C^{-1} = AB^{-1}$
(d) $C^{-1} = BA^{-1}$
(e) More than one of the above is true.
(f) Just because A and B have inverses, this doesn't mean that C has an inverse.

282. Let A be a 2×2 matrix. The inverse of $3A$ is

(a) $\frac{1}{9}A^{-1}$
(b) $\frac{1}{3}A^{-1}$
(c) A^{-1}
(d) $3A^{-1}$
(e) Not enough information is given.

283. If A is an invertible matrix, what else must be true?

(a) If $AB = C$ then $B = A^{-1}C$.

(b) A^2 is invertible.
(c) A^T is invertible.
(d) $5A$ is invertible.
(e) The reduced row echelon form of A is I.
(f) All of the above must be true.

Determinants

284. What is the determinant of $\begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$?

(a) 4
(b) 11
(c) 15
(d) 19

285. What is the determinant of $\begin{bmatrix} 5 & 1 & 0 \\ 1 & 3 & 2 \\ 0 & -1 & 1 \end{bmatrix}$?

(a) 0
(b) 15
(c) 24
(d) 26

286. What is the determinant of $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$?

(a) 0
(b) 9
(c) 15

287. What is the determinant of $\begin{bmatrix} 5 & 2 & -1 \\ 0 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$?

(a) 0
(b) 6
(c) 15
(d) 22

288. Which of the following matrices are not invertible?

(a) $\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & 3 \\ 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -3 \\ -2 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & -3 & 3 \\ -2 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

(e) More than one of the above
(f) All of the above have inverses

289. **True or False** $\det(A + B) = \det A + \det B$. Be prepared to support your answer either with a proof (at least for the 2×2 case) or a counterexample.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

290. **True or False** $\det(AB) = \det A \det B$. Be prepared to support your answer either with a proof (at least for the 2×2 case) or a counterexample.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

291. Suppose the determinant of a 2×2 matrix A is equal to 3. What is the determinant of A^{-1}?

(a) 1/3
(b) 3
(c) 9
(d) Not enough information is given.

292. Suppose the determinant of a 2×2 matrix A is equal to 3. What is the determinant of $5A$?

(a) 3
(b) 9
(c) 15
(d) 75
(e) Not enough information is given.

293. If A is a 2×2 matrix, then $\det(kA)$ is

(a) $k \det(A)$
(b) $2k \det(A)$
(c) $k^2 \det(A)$
(d) Not enough information is given.

294. Which of the following statements is true?

(a) If a square matrix has two identical rows then its determinant is zero.
(b) If the determinant of a matrix is zero, then the matrix has two identical rows.
(c) Both are true.
(d) Neither is true.

295. Suppose the determinant of matrix A is zero. How many solutions does the system $Ax = b$ have?

(a) 0
(b) 1
(c) Infinite
(d) Not enough information is given.

296. Suppose the determinant of matrix A is zero. How many solutions does the system $Ax = 0$ have?

   (a) 0
   (b) 1
   (c) Infinite
   (d) Not enough information is given.

Vector Spaces and Subspaces

297. Which property of vector spaces is not true for the following set?

$$\left\{ \begin{bmatrix} -2 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$$

   (a) Closure under vector addition
   (b) Existence of an additive identity
   (c) Existence of an additive inverse for each vector
   (d) None of the above

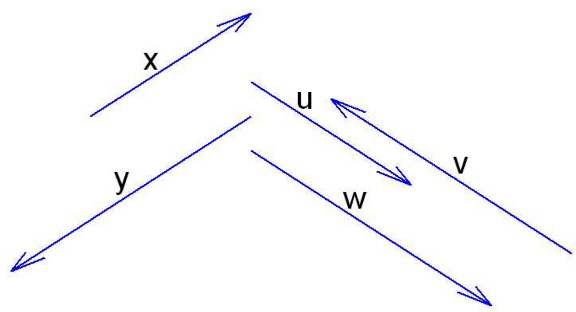
298. A vector subspace does *not* have to satisfy which of the following properties?

   (a) Associativity under vector addition
   (b) Existence of an additive identity
   (c) Commutativity under vector addition
   (d) A vector subspace must satisfy all of the above properties.
   (e) A vector subspace need not satisfy any of the above properties.

299. A vector space does *not* have to satisfy which of the following properties?

   (a) Closure under vector addition
   (b) Closure under scalar multiplication
   (c) Closure under vector multiplication
   (d) A vector subspace must satisfy all of the above properities.
   (e) A vector subspace need not satisfy any of the above properties.

300. Which of the following sets of vectors are contained within a proper subspace of $\Re^2$?



i. x, y

ii. u, v, w

iii. x, v

iv. y, u, w

(a) i, ii, iii, and iv
(b) ii, iii, and iv only
(c) i and ii only
(d) ii and iv only
(e) iii and iv only
(f) ii only

301. The set of all 2×2 matrices with determinant equal to zero is not a vector subspace. Why?

(a) 2×2 matrices are not vectors.
(b) With matrices, AB need not equal BA.
(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$ is not in the set.
(d) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is not in the set.
(e) None of the above

302. Which of the following sets of vectors is a basis for $\Re^3$?

(a) $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$
(b) $\{(1, 0, 1), (1, 1, 0), (1, 1, 1)\}$
(c) $\{(2, 0, 0), (0, 5, 0), (0, 0, 8)\}$
(d) All are bases for $\Re^3$.

303. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Which of the following sets has the same span as the set of all three vectors $\{v_1, v_2, v_3\}$?

(a) $\{v_1, v_2\}$

(b) $\{v_2, v_3\}$

(c) $\{v_1, v_3\}$

(d) None of the above

(e) More than one of the above

304. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Which of the following vectors is *not* in the subspace of $\Re^3$ spanned by $\{v_1, v_2, v_3\}$?

(a) $(1, 0, 0)$

(b) $(4, 1, 1)$

(c) $(3, 3, 6)$

(d) All of these are in the subspace of $\Re^3$ spanned by $\{v_1, v_2, v_3\}$.

305. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Geometrically, what is the subspace spanned by the set $\{v_1, v_2, v_3\}$?

(a) a point

(b) a line

(c) a plane

(d) a volume

(e) all of $\mathcal{R}^3$

306. Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, w = \begin{bmatrix} k \\ 2 \\ -3 \end{bmatrix}$. For how many values of k will the vector w be in the subspace spanned by $\{v_1, v_2, v_3\}$?

(a) No values of k - vector w will never be in this subspace

(b) Exactly one value of k will work.

(c) Any value of k will work.

307. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$, $w = \begin{bmatrix} k \\ 8 \\ 11 \end{bmatrix}$. For how many values of k will the vector w be in the subspace spanned by $\{v_1, v_2, v_3\}$?

(a) No values of k - vector w will never be in this subspace

(b) Exactly one value of k will work.

(c) Any value of k will work.

308. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$, $w = \begin{bmatrix} 1 \\ k \\ k \end{bmatrix}$. For how many values of k will the vector w be in the subspace spanned by $\{v_1, v_2, v_3\}$?

(a) No values of k - vector w will never be in this subspace

(b) Exactly one value of k will work.

(c) Any value of k will work.

Linear Combinations

309. If $u = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$, what is $2u - 3v$?

(a) $\begin{bmatrix} -4 \\ 4 \\ 23 \end{bmatrix}$

(b) $\begin{bmatrix} 8 \\ 4 \\ -7 \end{bmatrix}$

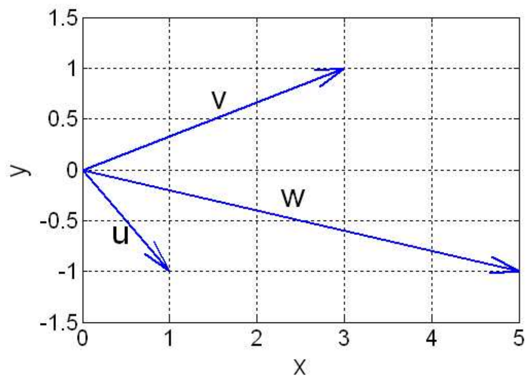
(c) $\begin{bmatrix} 8 \\ 4 \\ 23 \end{bmatrix}$

(d) $\begin{bmatrix} 7 \\ 6 \\ 2 \end{bmatrix}$

310. Write $z = \begin{bmatrix} -5 \\ 3 \\ 16 \end{bmatrix}$ as a linear combination of $x = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$ and $y = \begin{bmatrix} -3 \\ 2 \\ 6 \end{bmatrix}$.

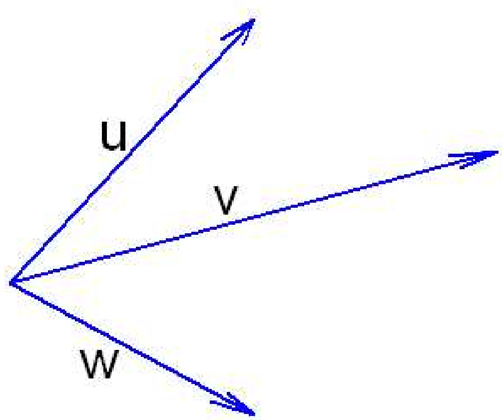
(a) $z = -5x$
(b) $z = -2x + y$
(c) $z = x + 2y$
(d) $z = 2x + y$
(e) z cannot be written as a linear combination of x and y.
(f) None of the above

311. Write the vector w as a linear combination of u and v.



(a) $w = 2u + v$
(b) $w = u + v$
(c) $w = -u + v$
(d) $w = u - v$
(e) w cannot be written as a linear combination of u and v.

312. Write the vector w as a linear combination of u and v.



(a) $w = 2u + v$

(b) $w = u + v$

(c) $w = -u + v$

(d) $w = u - v$

(e) w cannot be written as a linear combination of u and v.

313. Write $z = \begin{bmatrix} 2 \\ -2 \\ 6 \end{bmatrix}$ as a linear combination of $x = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ and $y = \begin{bmatrix} 4 \\ -5 \\ 7 \end{bmatrix}$.

(a) $z = x + y$

(b) $z = -x + y$

(c) $z = 3x + 2y$

(d) $z = -3x + y$

(e) z cannot be written as a linear combination of x and y.

(f) None of the above

314. Suppose we have the vectors $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$. Which of the following is *not* a linear combination of these?

(a) $\begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} 8 \\ 0 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 8 \\ 1 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

(e) $\begin{bmatrix} 40 \\ 5 \\ 15 \end{bmatrix}$

(f) More than one of the above is not a linear combination of the given vectors.

315. Suppose we have the vectors $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$. Which of the following is true?

(a) Every vector in $\Re^3$ can be written as a linear combination of these vectors.
(b) Some, but not all, vectors in $\Re^3$ can be written as a linear combination of these vectors.
(c) Every vector in $\Re^2$ can be written as a linear combination of these vectors.
(d) More than one of the above is true.
(e) None of the above are true.

316. Which of the following vectors can be written as a linear combination of the vectors (1, 0) and (0, 1)?

(a) $(2, 0)$
(b) $(-3, 1)$
(c) $(0.4, 3.7)$
(d) All of the above

317. How do you describe the set of all linear combinations of the vectors (1, 0) and (0, 1)?

(a) A point
(b) A line segment
(c) A line
(d) $\Re^2$
(e) $\Re^3$

318. Which of the following vectors can be written as a linear combination of the vectors (1, 0, 0), (0, 1, 0), and (0, 0, 1)?

(a) $(0, 2, 0)$
(b) $(-3, 0, 1)$
(c) $(0.4, 3.7, -1.5)$
(d) All of the above

319. How do you describe the set of all linear combinations of the vectors (1, 0, 0), (0, 1, 0), (0, 0, 1)?

(a) A point
(b) A line segment
(c) A line
(d) $\Re^2$
(e) $\Re^3$

320. How do you describe the set of all linear combinations of the vectors (1, 2, 0) and (-1, 1, 0)?

(a) A point
(b) A line
(c) A plane
(d) $\Re^2$
(e) $\Re^3$

321. Let z be any vector from $\Re^3$. If we have a set V of unknown vectors from $\Re^3$, how many vectors must be in V to guarantee that z can be written as a linear combination of the vectors in V?

(a) 2
(b) 3
(c) 4
(d) It is not possible to make such a guarantee.

322. Suppose y and z are both solutions to $Ax = b$. **True or False** All linear combinations of y and z also solve $Ax = b$. (You should be prepared to support your answer with either a proof or a counterexample.)

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

323. Suppose y and z are both solutions to $Ax = 0$. **True or False** All linear combinations of y and z also solve $Ax = 0$. (You should be prepared to support your answer with either a proof or a counterexample.)

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

324. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. In this context, what interpretation can be given to the vector $15s_1$?

(a) $15s_1$ shows the number of people that can be served with 15 gallons of vanilla ice cream.

(b) $15s_1$ shows the gallons of vanilla and chocolate ice cream sold by store 1 in 15 days.

(c) $15s_1$ gives the total revenue from selling 15 gallons of ice cream at store 1.

(d) $15s_1$ represents the number of days it will take to sell 15 gallons of ice cream at store 1.

325. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. The stores are run by different managers, and they are not always able to be open the same number of days in a month. If store 1 is open for c_1 days in March, and store 2 is open for c_2 days in March, which of the following represents the total sales of each flavor of ice cream between the two stores?

(a) $c_1 s_1 + c_2 s_2$

(b) $\begin{bmatrix} 5 & 6 \\ 8 & 10 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

(c) $\begin{bmatrix} 5c_1 \\ 8c_1 \end{bmatrix} + \begin{bmatrix} 6c_2 \\ 10c_2 \end{bmatrix}$

(d) All of the above

(e) None of the above

326. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. Lucinda is getting ready to close her ice cream parlors for the winter. She has a total of 39 gallons of vanilla ice cream in her warehouse, and 64 gallons of chocolate ice cream. She would like to distribute the ice cream to the two stores so that it is used up before the stores close for the winter. How much ice cream should she take to each store? The stores may stay open for different number of days, but no store may run out of ice cream before the end of the day on which it closes.

(a) Lucinda should take 3 gallons of each kind of ice cream to store 1 and 4 gallons of each kind to store 2.

(b) Lucinda should take 3 gallons of vanilla to each store and 4 gallons of chocolate to each store.

(c) Lucinda should take 15 gallons of vanilla and 24 gallons of chocolate to store 1, and she should take 24 gallons of vanilla and 40 gallons of chocolate to store 2.

(d) Lucinda should take 15 gallons of vanilla and 32 gallons of chocolate to store 1, and she should take 18 gallons of vanilla and 40 gallons of chocolate to store 2.

(e) This cannot be done unless ice cream is thrown out or a store runs out of ice cream before the end of the day.

Linear Independence

327. **True or False** The following vectors are linearly independent: (1,0,0), (0,0,2), (3,0,4)

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

328. Which set of vectors is linearly independent?

(a) $(2, 3), (8, 12)$

(b) $(1, 2, 3), (4, 5, 6), (7, 8, 9)$

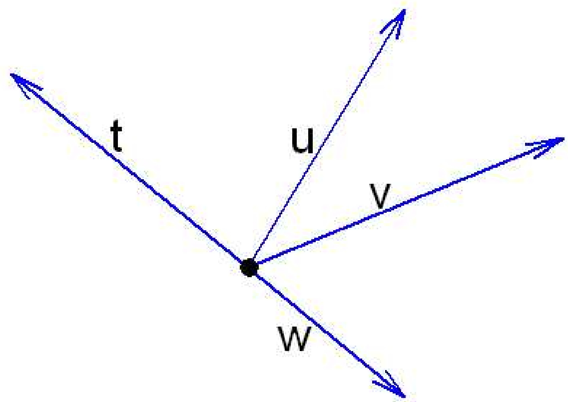
(c) $(-3, 1, 0), (4, 5, 2), (1, 6, 2)$

(d) None of these sets are linearly independent.

(e) Exactly two of these sets are linearly independent.

(f) All of these sets are linearly independent.

329. Which subsets of the set of the vectors shown below are linearly dependent?



(a) u, w

(b) t, w

(c) t, v

(d) t, u, v

(e) None of these sets are linearly dependent.

(f) More than one of these sets is linearly dependent.

330. Suppose you wish to determine whether a set of vectors is linearly independent. You form a matrix with those vectors as the columns, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. What do you decide?

(a) These vectors are linearly independent.

(b) These vectors are not linearly independent.

331. Suppose you wish to determine whether a set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent. You form the matrix $A = [v_1 v_2 v_3 v_4]$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write v_4 as a linear combination of v_1, v_2, and v_3. Which is a correct linear combination?

(a) $v_4 = v_1 + v_2$

(b) $v_4 = -v_1 - 2v_3$

(c) v_4 cannot be written as a linear combination of v_1, v_2, and v_3.

(d) We cannot determine the linear combination from this information.

332. Suppose you wish to determine whether a set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent. You form the matrix $A = [v_1 v_2 v_3 v_4]$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write v_3 as a linear combination of v_1, v_2, and v_4. Which is a correct linear combination?

(a) $v_3 = (1/2)v_1 - (1/2)v_4$

(b) $v_3 = (1/2)v_1 + (1/3)v_2$

(c) $v_3 = 2v_1 + 3v_2$

(d) $v_3 = -2v_1 - 3v_2$

(e) v_3 cannot be written as a linear combination of v_1, v_2, and v_4.

(f) We cannot determine the linear combination from this information.

333. Suppose you wish to determine whether a set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent. You form the matrix $A = [v_1 v_2 v_3 v_4]$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write v_2 as a linear combination of v_1, v_3, and v_4. Which is a correct linear combination?

(a) $v_2 = 3v_3 + v_4$

(b) $v_2 = -3v_3 - v_4$

(c) $v_2 = v_4 - 3v_3$

(d) $v_2 = -v_1 + v_4$

(e) v_2 cannot be written as a linear combination of v_1, v_3, and v_4.

(f) We cannot determine the linear combination from this information.

334. Are the vectors $\left\{ \begin{bmatrix} 1 \\ 4 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -14 \\ 13 \\ 7 \\ -19 \end{bmatrix} \right\}$ linearly independent?

(a) Yes, they are linearly independent.

(b) No, they are not linearly independent.

335. To determine whether a set of n vectors from $\Re^n$ is independent, we can form a matrix A whose columns are the vectors in the set and then put that matrix in reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

 (a) A row of all zeros.
 (b) A row that has all zeros except in the last position.
 (c) A column of all zeros.
 (d) An identity matrix.

336. To determine whether a set of fewer than n vectors from $\Re^n$ is independent, we can form a matrix A whose columns are the vectors in the set and then put that matrix in reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

 (a) An identity submatrix with zeros below it.
 (b) A row that has all zeros except in the last position.
 (c) A column that is not an identity matrix column.
 (d) A column of all zeros.

337. If the columns of A are not linearly independent, how many solutions are there to the system $Ax = 0$?

 (a) 0
 (b) 1
 (c) infinite
 (d) Not enough information is given.

338. **True or False** A set of 4 vectors from $\Re^3$ could be linearly independent.

 (a) True, and I am very confident
 (b) True, but I am not very confident
 (c) False, but I am not very confident
 (d) False, and I am very confident

339. **True or False** A set of 2 vectors from $\Re^3$ must be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

340. **True or False** A set of 3 vectors from $\Re^3$ could be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

341. **True or False** A set of 5 vectors from $\Re^4$ could be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

342. Which statement is equivalent to saying that v_1, v_2, and v_3 are linearly independent vectors?

(a) The only solution to $c_1v_1 + c_2v_2 + c_3v_3 = 0$ is $c_1 = c_2 = c_3 = 0$.
(b) v_3 cannot be written as a linear combination of v_1 and v_2.
(c) No vector is a multiple of any other.
(d) Exactly two of (a), (b), and (c) are true.
(e) All three statements are true.

Spanning Sets, Bases, and Dimension

343. Write $d = (3, -5, 10)$ as a linear combination of the vectors $a = (-1, 0, 3), b = (0, 1, 5)$, and $c = (4, -2, 0)$.

(a) $d = -3a - 5b + c$

(b) $d = 5a - b + 2c$

(c) $d = (10/3)a + (5/2)c$

(d) d cannot be written as a linear combination of a, b, and c.

344. Which of the following sets of vectors spans $\Re^3$?

i. $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$ ii. $\begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

iii. $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -2 \end{bmatrix}$ iv. $\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$

(a) i, ii, iii, and iv

(b) ii, iii, and iv only

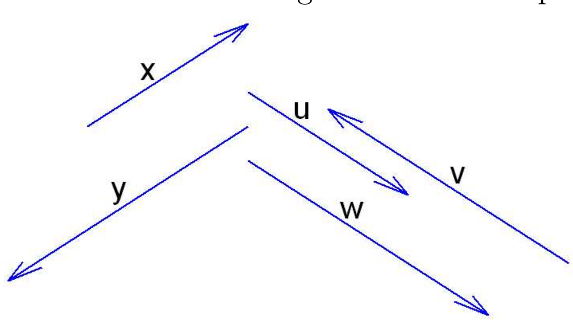
(c) ii and iii only

(d) i, ii and iii only

(e) iii and iv only

(f) ii only

345. Which of the following sets of vectors spans $\Re^2$?



i. x, y ii. u, v, w

iii. x, v iv. y, u, w

(a) i, ii, iii, and iv

(b) ii, iii, and iv only

(c) ii and iii only

(d) ii and iv only

(e) iii and iv only

(f) ii only

346. Which of the following sets of vectors forms a basis for $\Re^3$?

i. $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$

ii. $\begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

iii. $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -2 \end{bmatrix}$

iv. $\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$

(a) i, ii, iii, and iv

(b) ii, iii, and iv only

(c) ii and iii only

(d) i, ii and iii only

(e) iii and iv only

(f) ii only

347. Which of the following describes the subspace of $\Re^3$ spanned by the vectors
$\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$?

(a) A line

(b) A plane

(c) $\Re^2$

(d) All of $\Re^3$

(e) Both (b) and (c)

348. Which of the following describes a basis for a subspace V?

(a) A basis is a linearly independent spanning set for V.

(b) A basis is a minimal spanning set for V.

(c) A basis is a largest possible set of linearly independent vectors in V.

(d) All of the above

(e) Some of the above

(f) None of the above

349. Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 3 & -1 & 0 & 2 \end{bmatrix}$. The reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
What is the dimension of the column space of A?

(a) 1

(b) 2

(c) 3

(d) 4

350. Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 3 & -1 & 0 & 2 \end{bmatrix}$. The reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
Which columns would form a basis for the column space of A?

(a) All four

(b) The first three

(c) Any three

(d) Any two

351. Let $B = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. Which of the following describes the column space of B?

(a) The column space of B is all of $\Re^3$.

(b) The column space of B is a proper subset of $\Re^3$.

(c) The column space of B is $\Re^4$.

(d) The column space of B is a proper subset of $\Re^4$.

(e) None of the above

352. Let $B = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. What is the dimension of the column space of B?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4
(f) Infinite

353. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. What is the dimension of the nullspace of A?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4
(f) Infinite

354. Let A be an $n \times n$ matrix. If A is an invertible matrix, what else must be true?

(a) The columns of A form a basis of $\Re^n$.
(b) The rank of A is n.
(c) The dimension of the column space of A is n.
(d) The dimension of the null space of A is 0.
(e) All of the above must be true.
(f) More than one, but not all, of the above have to be true.

355. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

Standard Blend	Extra Wheat	Extra Soy	
0.5	0.8	0.3	whole wheat flour
0.5	0.2	0.7	soy flour

Do the column vectors in this table span $\Re^2$? Do they form a basis for $\Re^2$?

(a) Yes, they span $\Re^2$, and they form a basis.

(b) They do span $\Re^2$, but they do not form a basis.

(c) They do not span $\Re^2$, but they do form a basis for $\Re^2$.

(d) They do not span $\Re^2$, nor do they form a basis.

356. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

Standard Blend	Extra Wheat	Extra Soy	
0.5	0.8	0.3	whole wheat flour
0.5	0.2	0.7	soy flour

To save rent money, the store will be moving to a smaller space and will need to cut back on inventory. If possible, the manager would like to only stock two of these blends, and make the third from those as necessary. Which blends can be made from the others?

(a) Standard Blend can be made from Extra Wheat Blend and Extra Soy Blend.

(b) Extra Wheat Blend can be made from Standard Blend and Extra Soy Blend.

(c) Extra Soy Blend can be made from Standard Blend and Extra Wheat Blend.

(d) Any one blend can be made from the other two.

357. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

Standard Blend	Extra Wheat	Extra Soy	
0.5	0.8	0.3	whole wheat flour
0.5	0.2	0.7	soy flour

If the store continues to stock all three of these blends, which special-request blends could be made from these three?

(a) Any special request could be accomodated by mixing the right combination of these three blends.

(b) It would be possible to make any blend that is between 30% and 80% whole wheat.

(c) It would be possible to make a broader range of blends than what is described in answer (b), but there are still some blends that would not be possible.

(d) It would be possible to satisfy some special requests, but not all of the ones described in answer (b).

Fundamental Vector Subspaces

358. How many linearly independent columns are there in the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

   (a) 2
   (b) 1
   (c) 0

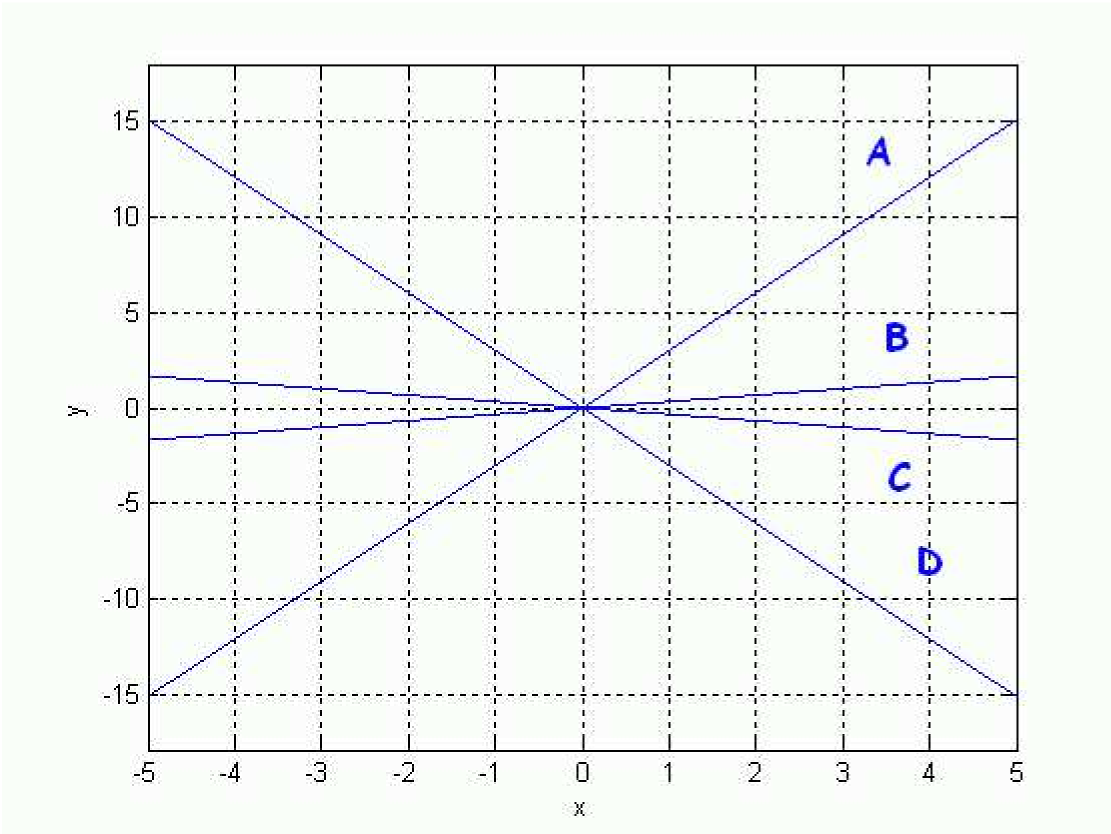
359. The *column space* of a matrix A is the set of vectors that can be created by taking all linear combinations of the columns of A. Is the vector $b = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$ in the column space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

   (a) Yes, since we can find a vector x so that $Ax = b$.
   (b) Yes, since $-2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} - 1 \begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$.
   (c) No, because there is no vector x so that $Ax = b$.
   (d) No, because we can't find c_1 and c_2 such that $c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$.
   (e) More than one of the above
   (f) None of the above

360. The column space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is

   (a) the set of all linear combinations of the columns of A.
   (b) a line in $\Re^2$.
   (c) the set of all multiples of the vector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.
   (d) All of the above
   (e) None of the above

361. Which line in the graph below represents the column space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?



(a) line A
(b) line B
(c) line C
(d) line D
(e) None of the above

362. How many solutions x are there to $Ax = 0$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) 0 solutions
(b) 1 solution
(c) 2 solutions
(d) Infinite number of solutions

363. The *null space* of a matrix A is the set of all vectors x that are solutions of $Ax = 0$. Which of the following vectors is in the null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) $x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

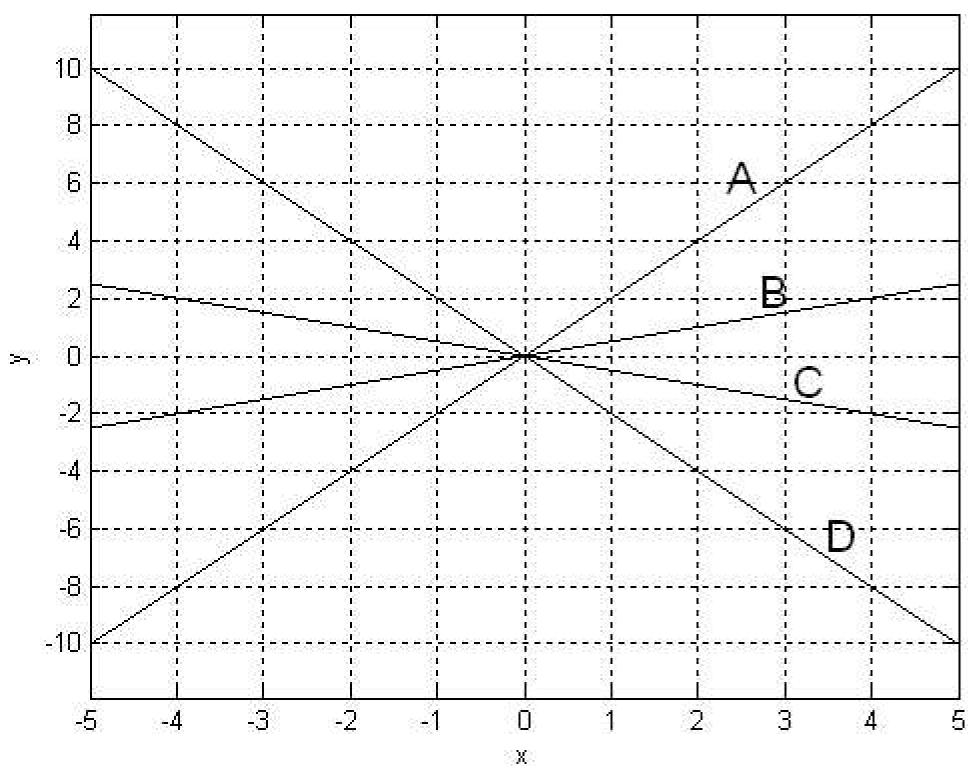
(b) $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(c) $x = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

(d) All of the above

(e) None of the above

364. Which line in the graph below represents the null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?



(a) line A

(b) line B

(c) line C

(d) line D

(e) None of the above

365. The *row space* of a matrix A is the set of vectors that can be created by taking all linear combinations of the rows of A. Which of the following vectors is in the row space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) $x = \begin{bmatrix} -2 & 4 \end{bmatrix}$
(b) $x = \begin{bmatrix} 4 & 8 \end{bmatrix}$
(c) $x = \begin{bmatrix} 0 & 0 \end{bmatrix}$
(d) $x = \begin{bmatrix} 8 & 4 \end{bmatrix}$
(e) More than one of the above
(f) None of the above

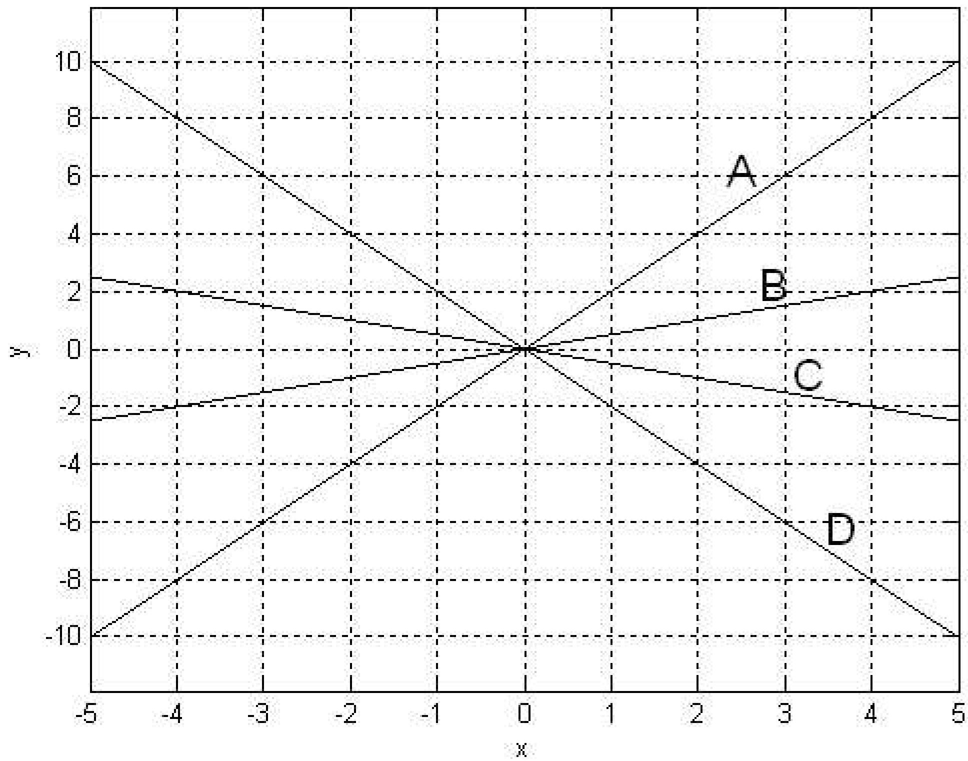
366. **True or False:** The row space of a matrix A is the same as the column space of A^T.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

367. The row space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ consists of

(a) All linear combinations of the columns of A^T.
(b) All multiples of the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.
(c) All linear combinations of the rows of A.
(d) All of the above
(e) None of the above

368. Which line in the graph below represents the row space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?



(a) line A
(b) line B
(c) line C
(d) line D
(e) None of the above

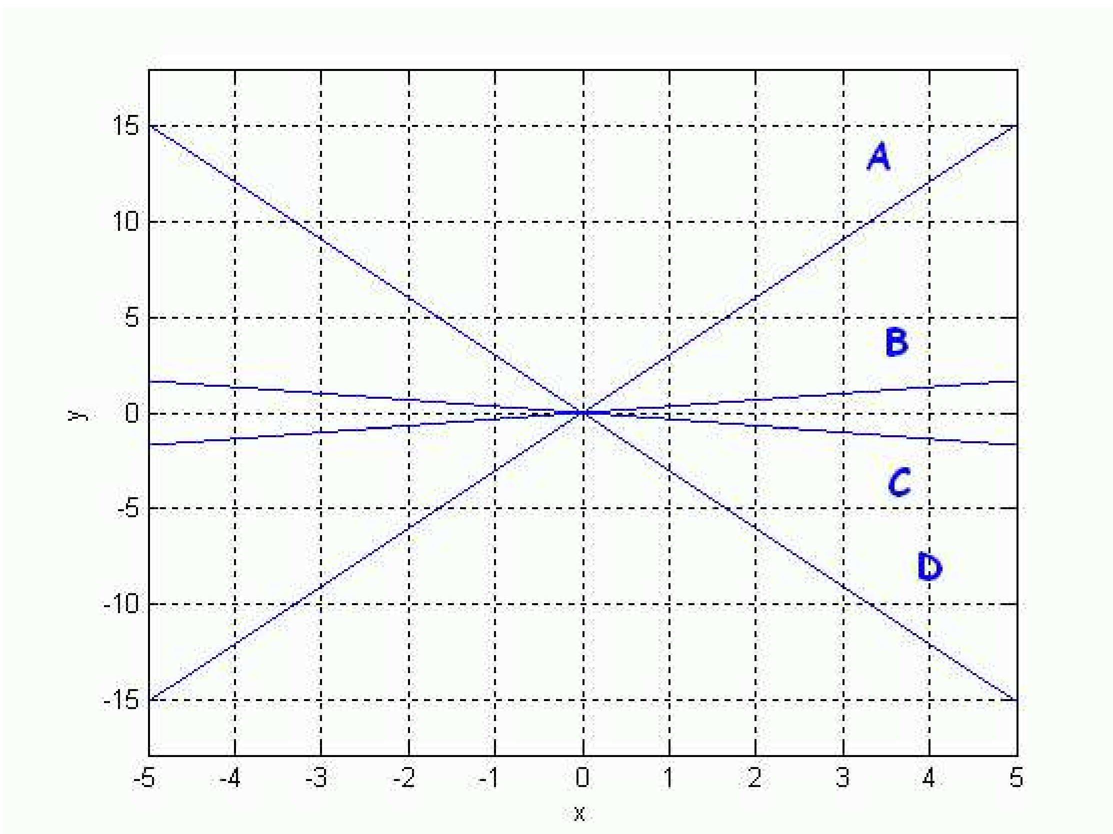
369. The *left null space* of a matrix A is the set of vectors x that solve $xA = 0$. Which of the following vectors is in the left null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) $x = \begin{bmatrix} -2 & 1 \end{bmatrix}$
(b) $x = \begin{bmatrix} -3 & 1 \end{bmatrix}$
(c) $x = \begin{bmatrix} 1 & -3 \end{bmatrix}$
(d) $x = \begin{bmatrix} 1 & -2 \end{bmatrix}$
(e) More than one of the above
(f) None of the above

370. **True or False:** Since $xA = 0$ can be rewritten as $A^T x^T = 0$, we can think of the left null space as the null space of A^T.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

371. Which line in the graph below represents the left null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?



(a) line A
(b) line B
(c) line C
(d) line D
(e) None of the above

372. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. Which of the following vectors are in the nullspace of A?

(a) $\begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 3 \\ -1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 3 \\ -1 \\ 3 \\ 2 \end{bmatrix}$

373. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. How many vectors are in the nullspace of A?

(a) Only one
(b) Probably more than one, but it's hard to say how many
(c) An infinite number

374. If A is an $m \times n$ matrix, then the column space of A is

(a) A subset of $\Re^m$ that may not include the origin.
(b) A subset of $\Re^m$ that includes the origin.
(c) A subset of $\Re^n$ that may not include the origin.
(d) A subset of $\Re^n$ that includes the origin.
(e) None of the above

375. If A is an $m \times n$ matrix, then the row space of A is

(a) A subset of $\Re^m$ that may not include the origin.
(b) A subset of $\Re^m$ that includes the origin.

(c) A subset of $\Re^n$ that may not include the origin.
(d) A subset of $\Re^n$ that includes the origin.
(e) None of the above

376. If A is an $m \times n$ matrix, then the null space of A is

(a) A subset of $\Re^m$ that may not include the origin.
(b) A subset of $\Re^m$ that includes the origin.
(c) A subset of $\Re^n$ that may not include the origin.
(d) A subset of $\Re^n$ that includes the origin.
(e) None of the above

377. If A is an $m \times n$ matrix, then the left null space of A is

(a) A subset of $\Re^m$ that may not include the origin.
(b) A subset of $\Re^m$ that includes the origin.
(c) A subset of $\Re^n$ that may not include the origin.
(d) A subset of $\Re^n$ that includes the origin.
(e) None of the above

378. Two vector spaces, V and W are *orthogonal complements* if and only if V is the set of all vectors which are orthogonal to every vector in W. Recall that for the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ the null space consists of all multiples of the vector $(-2, 1)$ and the left null space consists of all multiples of the vector $(-3, 1)$. Which of the following are true?

(a) The column space and null space are orthogonal complements.
(b) The column space and row space are orthogonal complements.
(c) The column space and left null space are orthogonal complements.
(d) None of the above

379. Recall that for the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ the null space consists of all multiples of the vector $(-2, 1)$ and the left null space consists of all multiples of the vector $(-3, 1)$. Which of the following vector subspaces are orthogonal complements?

(a) The row space and null space are orthogonal complements.
(b) The row space and column space are orthogonal complements.
(c) The row space and left null space are orthogonal complements.
(d) None of the above

Linearly Independent Sets

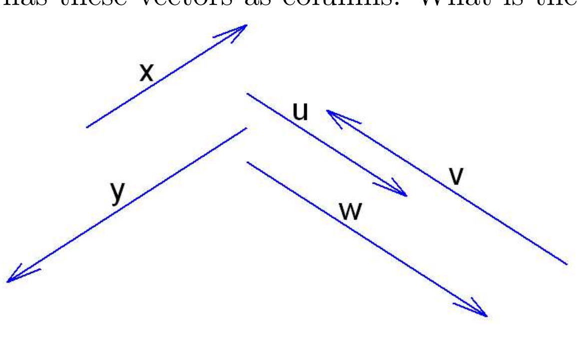
380. To determine whether a set S of vectors is linearly independent, you form a matrix which has those vectors as columns, and you calculate its reduced row echelon form. Suppose the resulting form is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. How many linearly independent vectors are in S?

    (a) 0
    (b) 1
    (c) 2
    (d) 3
    (e) 4

381. To determine whether a set S of vectors is linearly independent, you form a matrix which has those vectors as columns, and you calculate its reduced row echelon form. Suppose the resulting form is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Which of the following subsets of S are linearly independent?

    (a) The first, second, and third vectors
    (b) The first, second, and fourth vectors
    (c) The first, third, and fourth vectors
    (d) The second, third, and fourth vectors
    (e) All of the above
    (f) More than one, but not all, of the above

382. Consider the vectors x, y, u, v, and w in $\Re^2$ plotted below and form a matrix M which has these vectors as columns. What is the rank of this matrix?



(a) $\text{rank}(M) = 1$
(b) $\text{rank}(M) = 2$
(c) $\text{rank}(M) = 3$
(d) $\text{rank}(M) = 4$
(e) $\text{rank}(M) = 5$

383. To determine whether a set of vectors is linearly independent, you form a matrix which has those vectors as columns. If the matrix is square and its determinant is zero, what do you conclude?

(a) The vectors are linearly independent.
(b) The vectors are not linearly independent.
(c) This test is inconclusive, and further work must be done.

384. Which of the following expressions is a linear combination of the functions $f(t)$ and $g(t)$?

(a) $2f(t) + 3g(t) + 4$
(b) $f(t) - 2g(t) + t$
(c) $2f(t)g(t) - 3f(t)$
(d) $f(t) - g(t)$
(e) All of the above
(f) None of the above
(g) Some of the above

385. **True or False** The function $h(t) = 4 + 3t$ is a linear combination of the functions $f(t) = (1+t)^2$ and $g(t) = 2 - t - 2t^2$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

386. **True or False** The function $h(t) = \sin(t+2)$ is a linear combination of the functions $f(t) = \sin t$ and $g(t) = \cos t$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

387. **True or False** $h(t) = t^2$ is a linear combination of $f(t) = (1-t)^2$ and $g(t) = (1+t)^2$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

388. Let $y_1(t) = \sin(2t)$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?

(a) $y_2(t) = \sin(t)\cos(t)$

(b) $y_2(t) = 2\sin(2t)$

(c) $y_2(t) = \cos(2t - \pi/2)$

(d) $y_2(t) = \sin(-2t)$

(e) All of the above

(f) None of the above

389. Let $y_1(t) = e^{2t}$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?

(a) $y_2(t) = e^{-2t}$

(b) $y_2(t) = te^{2t}$

(c) $y_2(t) = 1$

(d) $y_2(t) = e^{3t}$

(e) All of the above

(f) None of the above

390. The functions $y_1(t)$ and $y_2(t)$ are linearly independent on the interval $a < t < b$ if

(a) for some constant k, $y_1(t) = ky_2(t)$ for $a < t < b$.

(b) there exists some $t_0 \in (a, b)$ and some constants c_1 and c_2 such that $c_1y_1(t_0) + c_2y_2(t_0) \neq 0$.

(c) the equation $c_1y_1(t) + c_2y_2(t) = 0$ holds for all $t \in (a, b)$ only if $c_1 = c_2 = 0$.

(d) the ratio $y_1(t)/y_2(t)$ is a constant function.

(e) All of the above

(f) None of the above

391. The functions $y_1(t)$ and $y_2(t)$ are linearly dependent on the interval $a < t < b$ if

(a) there exist two constants c_1 and c_2 such that $c_1y_1(t)+c_2y_2(t) = 0$ for all $a < t < b$.

(b) there exist two constants c_1 and c_2, not both 0, such that $c_1y_1(t) + c_2y_2(t) = 0$ for all $a < t < b$.

(c) for each t in (a, b), there exists constants c_1 and c_2 such that $c_1y_1(t)+c_2y_2(t) = 0$.

(d) for some $a < t_0 < b$, the equation $c_1y_1(t_0) + c_2y_2(t_0) = 0$ can only be true if $c_1 = c_2 = 0$.

(e) All of the above

(f) None of the above

Chapter 4: Higher-Order Linear Differential Equations

Second Order Differential Equations: Oscillations

392. A branch sways back and forth with position $f(t)$. Studying its motion you find that its acceleration is proportional to its position, so that when it is 8 cm to the right, it will accelerate to the left at a rate of 2 cm/s^2. Which differential equation describes the motion of the branch?

(a) $\frac{d^2f}{dt^2} = 8f$

(b) $\frac{d^2f}{dt^2} = -4f$

(c) $\frac{d^2f}{dt^2} = -2$

(d) $\frac{d^2f}{dt^2} = \frac{f}{4}$

(e) $\frac{d^2f}{dt^2} = -\frac{f}{4}$

393. The differential equation $\frac{d^2f}{dt^2} = -0.1f + 3$ is solved by a function $f(t)$ where f is in feet and t is in minutes. What units does the number 3 have?

(a) feet

(b) minutes

(c) per minute

(d) per minute^2

(e) feet per minute^2

(f) no units

394. The differential equation $y'' = 7y$ is solved by a function $y(t)$ where y is in meters and t is in seconds. What units does the number 7 have?

(a) meters

(b) seconds

(c) per second

(d) per second^2

(e) meters per second^2

(f) no units

395. A differential equation is solved by the function $y(t) = 3\sin 2t$ where y is in meters and t is in seconds. What units do the numbers 3 and 2 have?

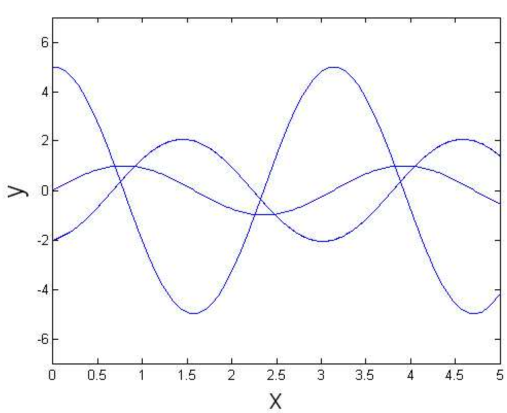
(a) 3 is in meters, 2 is in seconds

(b) 3 is in meters, 2 is in per second

(c) 3 is in meters per second, 2 has no units

(d) 3 is in meters per second, 2 is in seconds

396. Three different functions are plotted below. Could these all be solutions of the same second order differential equation?



(a) Yes
(b) No
(c) Not enough information is given.

397. Which of the following is not a solution of $y'' + ay = 0$ for some value of a?

(a) $y = 4\sin 2t$
(b) $y = 8\cos 3t$
(c) $y = 2e^{2t}$
(d) all are solutions

398. The functions below are solutions of $y'' + ay = 0$ for different values of a. Which represents the largest value of a?

(a) $y(t) = 100\sin 2\pi t$
(b) $y(t) = 25\cos 6\pi t$
(c) $y(t) = 0.1\sin 50t$
(d) $y(t) = 3\sin 2t + 8\cos 2t$

399. Each of the differential equations below represents the motion of a mass on a spring. If the mass is the same in each case, which spring is stiffer?

(a) $s'' + 8s = 0$
(b) $s'' + 2s = 0$
(c) $2s'' + s = 0$
(d) $8s'' + s = 0$

400. The motion of a mass on a spring follows the equation $mx'' = -kx$ where the displacement of the mass is given by $x(t)$. Which of the following would result in the highest frequency motion?

(a) $k = 6$, $m = 2$
(b) $k = 4$, $m = 4$
(c) $k = 2$, $m = 6$
(d) $k = 8$, $m = 6$
(e) All frequencies are equal

401. Each of the differential equations below represents the motion of a mass on a spring. Which system has the largest maximum velocity?

(a) $2s'' + 8s = 0, s(0) = 5, s'(0) = 0$
(b) $2s'' + 4s = 0, s(0) = 7, s'(0) = 0$
(c) $s'' + 4s = 0, s(0) = 10, s'(0) = 0$
(d) $8s'' + s = 0, s(0) = 20, s'(0) = 0$

402. Which of the following is not a solution of $\frac{d^2y}{dt^2} = -ay$ for some positive value of a?

(a) $y = 2\sin 6t$
(b) $y = 4\cos 5t$
(c) $y = 3\sin 2t + 8\cos 2t$
(d) $y = 2\sin 3t + 2\cos 5t$

403. Which function does not solve both $z' = 3z$ and $z'' = 9z$?

(a) $z = 7e^{3t}$
(b) $z = 0$
(c) $z = 12e^{-3t}$
(d) $z = -6e^{3t}$
(e) all are solutions to both

404. How are the solutions of $y'' = \frac{1}{4}y$ different from solutions of $y' = \frac{1}{2}y$?

(a) The solutions of $y'' = \frac{1}{4}y$ grow half as fast as solutions of $y' = \frac{1}{2}y$.
(b) The solutions of $y'' = \frac{1}{4}y$ include decaying exponentials.
(c) The solutions of $y'' = \frac{1}{4}y$ include sines and cosines.
(d) None of the above

405. How are the solutions of $y'' = -\frac{1}{4}y$ different from solutions of $y'' = -\frac{1}{2}y$?

(a) The solutions of $y'' = -\frac{1}{4}y$ oscillate twice as fast as the solutions of $y'' = -\frac{1}{2}y$.
(b) The solutions of $y'' = -\frac{1}{4}y$ have a period which is twice as long as the solutions of $y'' = -\frac{1}{2}y$.
(c) The solutions of $y'' = -\frac{1}{4}y$ have a smaller maximum value than the solutions of $y'' = -\frac{1}{2}y$.

(d) More than one of the above is true.

(e) None of the above are true.

406. What function solves the equation $y'' + 10y = 0$?

(a) $y = 10 \sin 10t$

(b) $y = 60 \cos \sqrt{10}t$

(c) $y = \sqrt{10}e^{-10t}$

(d) $y = 20e^{\sqrt{10t}}$

(e) More than one of the above

407. We know that the solution of a differential equation is of the form $y = A \sin 3x + B \cos 3x$. Which of the following would tell us that $A = 0$?

(a) $y(0) = 0$

(b) $y'(0) = 0$

(c) $y(1) = 0$

(d) none of the above

408. We know that the solutions to a differential equation are of the form $y = Ae^{3x} + Be^{-3x}$. If we know that $y = 0$ when $x = 0$, this means that

(a) $A = 0$

(b) $B = 0$

(c) $A = -B$

(d) $A = B$

(e) none of the above

409. An ideal spring produces an acceleration that is proportional to the displacement, so $my'' = -ky$ for some positive constant k. In the lab, we find that a mass is held on an imperfect spring: As the mass gets farther from equilibrium, the spring produces a force stronger than an ideal spring. Which of the following equations could model this scenario?

(a) $my'' = ky^2$

(b) $my'' = -k\sqrt{y}$

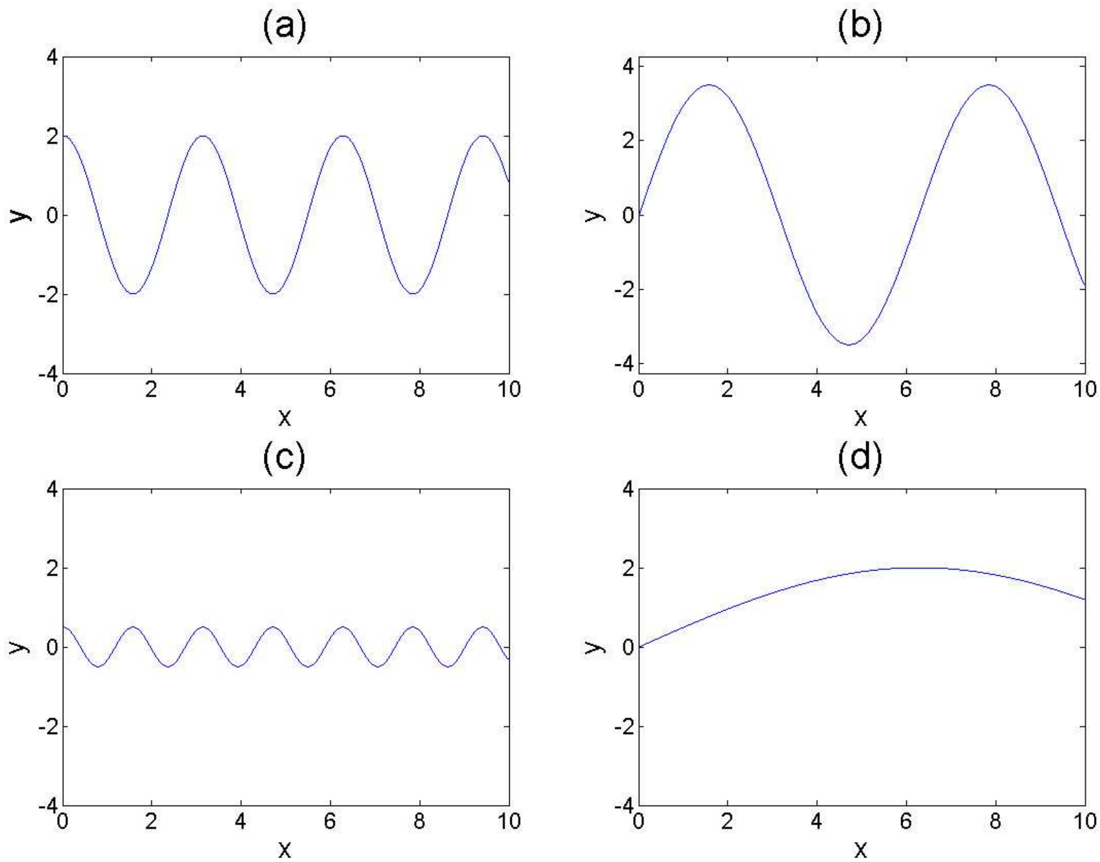
(c) $my'' = -k|y|$

(d) $my'' = -ky^3$

(e) $my'' = -ke^{-y}$

(f) None of the above

410. The functions plotted below are solutions of $y'' = -ay$ for different positive values of a. Which case corresponds to the largest value of a?



411. The motion of a child bouncing on a trampoline is modeled by the equation $p''(t) + 3p(t) = 6$ where p is in inches and t is in seconds. Suppose we want the position function to be in feet instead of inches. How does this change the differential equation?

(a) There is no change

(b) $p''(t) + 3p(t) = 0.5$

(c) $p''(t) + 3p(t) = 72$

(d) $144p''(t) + 3p(t) = 3$

(e) $p''(t) + 36p(t) = 3$

(f) $144p''(t) + 36p(t) = 3$

412. A float is bobbing up and down on a lake, and the distance of the float from the lake floor follows the equation $2d'' + 5d - 30 = 0$, where $d(t)$ is in feet and t is in seconds. At what distance from the lake floor could the float reach equilibrium?

(a) 2 feet
(b) 5 feet
(c) 30 feet
(d) 6 feet
(e) 15 feet
(f) No equilibrium exists.

Second Order Differential Equations: Damping

413. Which of the following equations is not equivalent to $y'' + 3y' + 2y = 0$?

(a) $2y'' + 6y' + 4y = 0$
(b) $y'' = 3y' + 2y$
(c) $-12y'' = 36y' + 24y$
(d) $3y'' = -9y' - 6y$
(e) All are equivalent.

414. Which of the following equations is equivalent to $y'' + \frac{2}{t}y' + \frac{3}{t^2}y = 0$?

(a) $t^2y'' + 2ty' + 3y = 0$
(b) $y'' + 2y' + 3y = 0$
(c) $t^2y'' + \frac{t}{2}y' + \frac{1}{3}y = 0$
(d) None are equivalent.

415. The motion of a child bouncing on a trampoline is modeled by the equation $p''(t) + 3p'(t)+8p(t) = 0$ where p is in feet and t is in seconds. What will the child's acceleration be if he is 3 feet below equilibrium and moving up at 6 ft/s?

(a) 6 ft/s^2 up
(b) 6 ft/s^2 down
(c) 42 ft/s^2 up
(d) 42 ft/s^2 down

(e) 39 ft/s^2 down

(f) None of the above

416. The motion of a child on a trampoline is modeled by the equation $p''(t)+2p'(t)+3p(t) = 0$ where p is in feet and t is in seconds. Suppose we want the position function to be in inches instead of feet. How does this change the differential equation?

(a) There is no change

(b) $p''(t) + 24p'(t) + 36p(t) = 0$

(c) $12p''(t) + 2p'(t) + 36p(t) = 0$

(d) $144p''(t) + 24p'(t) + 3p(t) = 0$

(e) None of the above

417. A hydrogen atom is bound to a large molecule, and its distance from the molecule follows the equation $d'' + 4d' + 8d - 6 = 0$ where d is in picometers. At what distance from the molecule will the atom reach equilibrium?

(a) $d = 6$ pm.

(b) $d = \frac{3}{4}$ pm.

(c) $d = \frac{6}{13}$ pm

(d) No equilibrium exists.

418. When the space shuttle re-enters the Earth's atmosphere, the air resistance produces a force proportional to its velocity squared, and gravity produces an approximately constant force. Which of the following equations might model its position $p(t)$ if a and b are positive constants?

(a) $p'' + a(p')^2 + b = 0$

(b) $p'' - a(p')^2 + b = 0$

(c) $p'' + a(p')^2 + bp = 0$

(d) $p'' - a(p')^2 + bp = 0$

(e) None of the above

419. The differential equation $m\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + ky = 0$ with positive parameters m, γ, and k is often used to model the motion of a mass on a spring with a damping force. If γ was negative, what would this mean?

(a) This would be like a negative friction, making the oscillations speed up over time.

(b) This would be like a negative spring, that would push the object farther and farther from equilibrium.

(c) This would be like a negative mass, so that the object would accelerate in the opposite direction that the forces were pushing.

(d) None of the above

420. Test the following functions to see which is a solution to $y'' + 4y' + 3y = 0$.

(a) $y = e^{2t}$

(b) $y = e^{t}$

(c) $y = e^{-t}$

(d) $y = e^{-2t}$

(e) None of these are solutions.

(f) All are solutions.

421. Test the following functions to see which is a solution to $\frac{d^2g}{dx^2} + 2\frac{dg}{dx} + 2g = 0$.

(a) $g = e^{x}$

(b) $g = \sin x$

(c) $g = e^{-x}\sin x$

(d) None of these are solutions.

422. Suppose we want to solve the differential equation $y'' + ay' + by = 0$ and we conjecture that our solution is of the form $y = Ce^{rt}$. What equation do we get if we test this solution and simplify the result?

(a) $1 + ar + br^2 = 0$

(b) $C^2r^2 + Cr + c = 0$

(c) $Ce^{rt} + aCe^{rt} + bCe^{rt} = 0$

(d) $r^2 + ar + b = 0$

(e) None of the above

423. Suppose we want to solve the differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 0$ and we conjecture that our solution is of the form $y = Ce^{rt}$. Solve the characteristic equation to determine what values of r satisfy the differential equation.

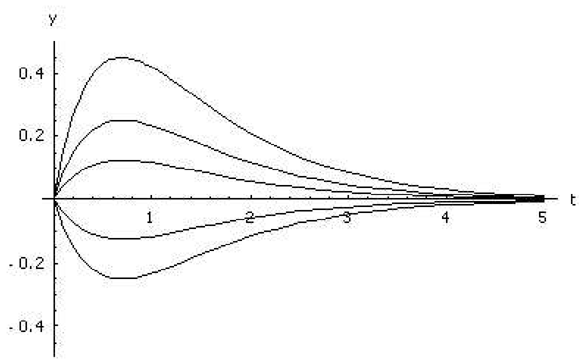
(a) $r = -2, -8$

(b) $r = -1, -4$
(c) $r = -3/2, +3/2$
(d) $r = 1, 4$
(e) None of the above

424. Find the general solution to $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$.

(a) $y(t) = C_1e^{-t/2} + C_2e^{t/2}$
(b) $y(t) = C_1e^{-2t} + C_2e^{-t}$
(c) $y(t) = C_1e^{-2t} + C_2e^{t}$
(d) $y(t) = -2C_1e^{-2t} - C_2e^{-t}$
(e) None of the above

425. The graph below has five trajectories, call the top one y_1, the one below it y_2, down to the lowest one y_5. Which of these could be a solution of $y'' + 3y' + 2y = 0$ with $y(0) = 0$ and $y'(0) = 1$?



(a) y_1
(b) y_2
(c) y_3
(d) y_4
(e) y_5

426. What is the general solution to $f'' + 2f' + 2f = 0$?

(a) $f(x) = C_1e^{-x/2}\cos x + C_2e^{-x/2}\sin x$
(b) $f(x) = C_1e^{-x}\cos x + C_2e^{-x}\sin x$
(c) $f(x) = C_1e^{-x}\cos\frac{x}{2} + C_2e^{-x}\sin\frac{x}{2}$

(d) $f(x) = C_1 + C_2e^{-2x}$
(e) None of the above

427. The harmonic oscillator modeled by $mx'' + bx' + kx = 0$ with parameters $m = 1$, $k = 2$, and $b = 1$ is underdamped and thus oscillates. What is the period of the oscillations?

(a) $2\pi/\sqrt{7}$
(b) $4\pi/\sqrt{7}$
(c) $\sqrt{7}/2\pi$
(d) $\sqrt{7}/4\pi$
(e) None of the above.

428. A harmonic oscillator is modeled by $mx'' + bx' + kx = 0$. If we increase the parameter m slightly, what happens to the period of oscillation?

(a) The period gets larger.
(b) The period gets smaller.
(c) The period stays the same.

429. A harmonic oscillator is modeled by $mx'' + bx' + kx = 0$. If we increase the parameter k slightly, what happens to the period of oscillation?

(a) The period gets larger.
(b) The period gets smaller.
(c) The period stays the same.

430. A harmonic oscillator is modeled by $mx'' + bx' + kx = 0$. If we increase the parameter b slightly, what happens to the period of oscillation?

(a) The period gets larger.
(b) The period gets smaller.
(c) The period stays the same.

431. Classify the harmonic oscillator described below:

$$3\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = 0.$$

(a) underdamped
(b) overdamped
(c) critically damped
(d) no damping

432. Does the harmonic oscillator described below oscillate?

$$3\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = 0.$$

(a) Yes.
(b) No.

Linear Combinations and Independence of Functions

433. Which of the following expressions is a linear combination of the functions $f(t)$ and $g(t)$?

(a) $2f(t) + 3g(t) + 4$
(b) $f(t) - 2g(t) + t$
(c) $2f(t)g(t) - 3f(t)$
(d) $f(t) - g(t)$
(e) All of the above
(f) None of the above

434. **True or False** The function $h(t) = 4 + 3t$ is a linear combination of the functions $f(t) = (1+t)^2$ and $g(t) = 2 - t - 2t^2$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

435. **True or False** The function $h(t) = \sin(t+2)$ is a linear combination of the functions $f(t) = \sin t$ and $g(t) = \cos t$.

(a) True, and I am very confident

(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

436. **True or False** $h(t) = t^2$ is a linear combination of $f(t) = (1-t)^2$ and $g(t) = (1+t)^2$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

437. Let $y_1(t) = \sin(2t)$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?

(a) $y_2(t) = \sin(t)\cos(t)$
(b) $y_2(t) = 2\sin(2t)$
(c) $y_2(t) = \cos(2t - \pi/2)$
(d) $y_2(t) = \sin(-2t)$
(e) All of the above
(f) None of the above

438. Let $y_1(t) = e^{2t}$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?

(a) $y_2(t) = e^{-2t}$
(b) $y_2(t) = te^{2t}$
(c) $y_2(t) = 1$
(d) $y_2(t) = e^{3t}$
(e) All of the above
(f) None of the above

439. The functions $y_1(t)$ and $y_2(t)$ are linearly independent on the interval $a < t < b$ if

(a) for some constant k, $y_1(t) = ky_2(t)$ for $a < t < b$.
(b) there exists some $t_0 \in (a, b)$ and some constants c_1 and c_2 such that $c_1y_1(t_0) + c_2y_2(t_0) \neq 0$.

(c) the equation $c_1y_1(t) + c_2y_2(t) = 0$ holds for all $t \in (a, b)$ only if $c_1 = c_2 = 0$.

(d) the ratio $y_1(t)/y_2(t)$ is a constant function.

(e) All of the above

(f) None of the above

440. The functions $y_1(t)$ and $y_2(t)$ are linearly dependent on the interval $a < t < b$ if

(a) there exist two constants c_1 and c_2 such that $c_1y_1(t)+c_2y_2(t) = 0$ for all $a < t < b$.

(b) there exist two constants c_1 and c_2, not both 0, such that $c_1y_1(t) + c_2y_2(t) = 0$ for all $a < t < b$.

(c) for each t in (a, b), there exists constants c_1 and c_2 such that $c_1y_1(t)+c_2y_2(t) = 0$.

(d) for some $a < t_0 < b$, the equation $c_1y_1(t_0) + c_2y_2(t_0) = 0$ can only be true if $c_1 = c_2 = 0$.

(e) All of the above

(f) None of the above

441. The functions $y_1(t)$ and $y_2(t)$ are both solutions of a certain second-order linear homogeneous differential equation with continuous coefficients for $a < t < b$. Which of the following statements are true?

(i) The general solution to the ODE is $y(t) = c_1y_1(t) + c_2y_2(t)$, $a < t < b$.

(ii) $y_1(t)$ and $y_2(t)$ must be linearly independent, since they both are solutions.

(iii) $y_1(t)$ and $y_2(t)$ may be linearly dependent, in which case we do not know enough information to write the general solution.

(iv) The Wronskian of $y_1(t)$ and $y_2(t)$ must be nonzero for these functions.

(a) Only (i) and (ii) are true.

(b) Only (i) is true.

(c) Only (ii) and (iv) are true.

(d) Only (iii) is true.

(e) None are true.

442. Can the functions $y_1(t) = t$ and $y_2(t) = t^2$ be a linearly independent pair of solutions for an ODE of the form

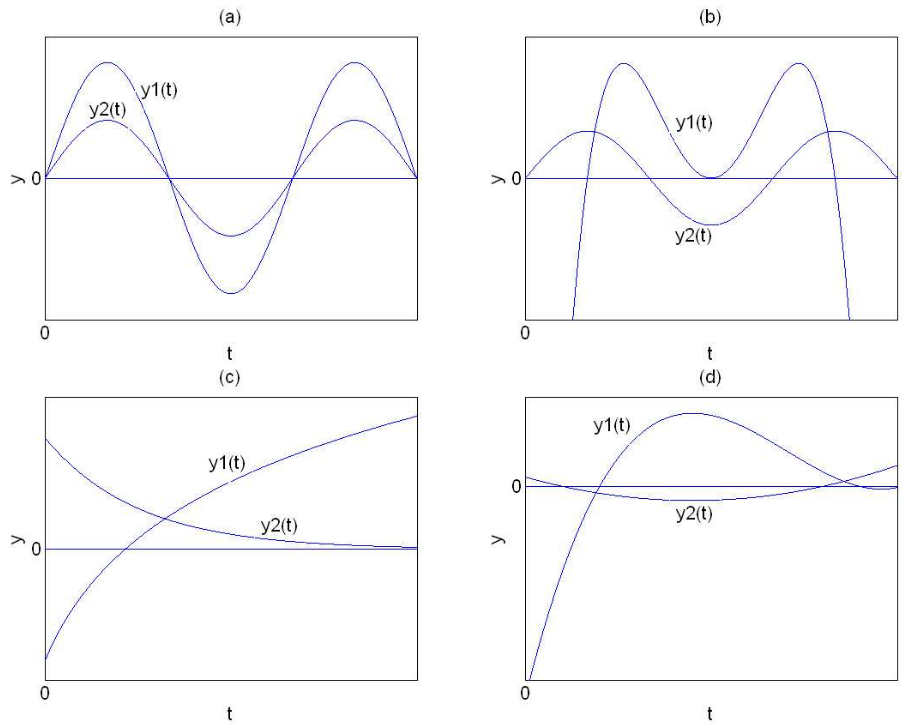
$$y'' + p(t)y' + q(t)y = 0 \qquad -1 \leq t \leq 1$$

where $p(t)$ and $q(t)$ are continuous functions?

(a) Yes

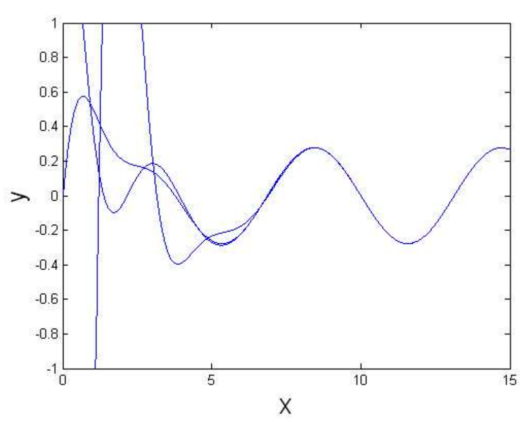
(b) No

443. Which pair of functions whose graphs are shown below could be linearly independent pairs of solutions to a second-order linear homogeneous differential equation?



Second Order Differential Equations: Forcing

444. The three functions plotted below are all solutions of $y'' + ay' + 4y = \sin x$. Is a positive or negative?



(a) a is positive.

(b) a is negative.

(c) $a = 0$.

(d) Not enough information is given.

445. If we conjecture the function $y(x) = C_1 \sin 2x + C_2 \cos 2x + C_3$ as a solution to the differential equation $y'' + 4y = 8$, which of the constants is determined by the differential equation?

(a) C_1

(b) C_2

(c) C_3

(d) None of them are determined.

446. What will the solutions of $y'' + ay' + by = c$ look like if b is negative and a is positive.

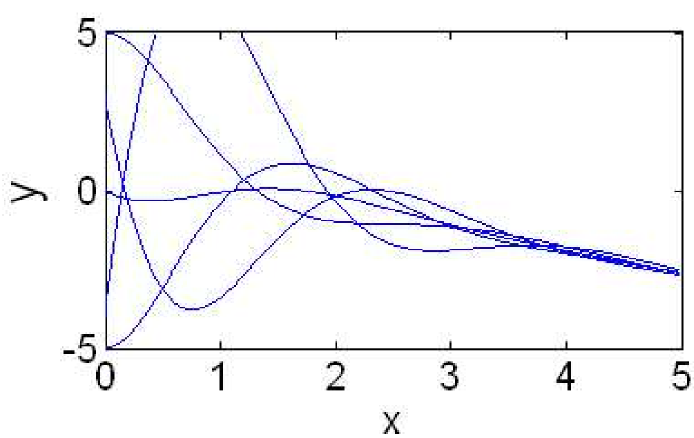
(a) Solutions will oscillate at first and level out at a constant.

(b) Solutions will grow exponentially.

(c) Solutions will oscillate forever.

(d) Solutions will decay exponentially.

447. The functions plotted below are solutions to which of the following differential equations?



(a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 3 - 3x$

(b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 3e^{2x}$

(c) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = \sin\frac{2\pi}{9}x$

(d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3x - 4$

(e) None of the above

448. The general solution to $f'' + 7f' + 12f = 0$ is $f(t) = C_1e^{-3t} + C_2e^{-4t}$. What should we conjecture as a particular solution to $f'' + 7f' + 12f = 5e^{-2t}$?

(a) $f(t) = Ce^{-4t}$

(b) $f(t) = Ce^{-3t}$

(c) $f(t) = Ce^{-2t}$

(d) $f(t) = C\cos 2t$

(e) None of the above

449. The general solution to $f'' + 7f' + 12f = 0$ is $f(t) = C_1e^{-3t} + C_2e^{-4t}$. What is a particular solution to $f'' + 7f' + 12f = 5e^{-6t}$?

(a) $f(t) = \frac{5}{6}e^{-6t}$

(b) $f(t) = \frac{5}{31}e^{-6t}$

(c) $f(t) = \frac{5}{20}e^{-6t}$

(d) $f(t) = e^{-3t}$

(e) None of the above

450. To find a particular solution to the differential equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t,$$

we replace it with a new differential equation that has been "complexified." What is the new differential equation to which we will find a particular solution?

(a) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{2it}$

(b) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{3it}$

(c) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$

(d) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-2it}$

(e) $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y = e^{-it}$

(f) None of the above.

451. To solve the differential equation $\dfrac{d^2y}{dt^2}+3\dfrac{dy}{dt}+2y = e^{it}$, we make a guess of $y_p(t) = Ce^{it}$. What equation results when we evaluate this in the differential equation?

(a) $-Ce^{it} + 3Cie^{it} + 2Ce^{it} = Ce^{it}$

(b) $-Ce^{it} + 3Cie^{it} + 2Ce^{it} = e^{it}$

(c) $Cie^{it} + 3Ce^{it} + 2Ce^{it} = e^{it}$

(d) $-Ce^{it} + 3Ce^{it} + 2Ce^{it} = Ce^{it}$

452. To solve the differential equation $\dfrac{d^2y}{dt^2}+3\dfrac{dy}{dt}+2y = e^{it}$, we make a guess of $y_p(t) = Ce^{it}$. What value of C makes this particular solution work?

(a) $C = \dfrac{1+3i}{10}$

(b) $C = \dfrac{1-3i}{10}$

(c) $C = \dfrac{1+3i}{\sqrt{10}}$

(d) $C = \dfrac{1-3i}{\sqrt{10}}$

453. In order to find a particular solution to $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y = \cos t$, do we want the real part or the imaginary part of the particular solution $y_p(t) = Ce^{it}$ that solved the complexified equation $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y = e^{it}$?

(a) Real part

(b) Imaginary part

(c) Neither, we need the whole solution to the complexified equation.

454. What is a particular solution to $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y = \cos t$?

(a) $y_p(t) = \dfrac{3}{10}\cos t + \dfrac{1}{10}\sin t$

(b) $y_p(t) = \frac{-3}{10}\cos t + \frac{1}{10}\sin t$

(c) $y_p(t) = \frac{1}{10}\cos t + \frac{-3}{10}\sin t$

(d) $y_p(t) = \frac{1}{10}\cos t + \frac{3}{10}\sin t$

Beats and Resonance

455. Which of the following forced 2nd order equations has solutions exhibiting *resonance*?

(a) $y'' + y = \cos(t)$

(b) $y'' + y = 2\cos(t)$

(c) $y'' + y = -2\cos(t)$

(d) All of the above

(e) None of the above

456. Which of the following forced 2nd order equations has solutions exhibiting *resonance*?

(a) $2y'' + y = \cos(t)$

(b) $2y'' + 4y = 2\cos(2t)$

(c) $4y'' + y = -2\cos(t/2)$

(d) All of the above

(e) None of the above

457. Which of the following forced 2nd order equations has solutions exhibiting *resonance*?

(a) $y'' + 2y = 10\cos(2t)$

(b) $y'' + 4y = 8\cos(2t)$

(c) $y'' + 2y = 6\cos(4t)$

(d) All of the above

(e) None of the above

458. Which of the following forced 2nd order equations has solutions clearly exhibiting *beats*?

(a) $y'' + 3y = 10\cos(2t)$

(b) $y'' + 1y = 2\cos(2t)$
(c) $y'' + 9y = 1\cos(3t)$
(d) All of the above
(e) None of the above

459. The differential equation $y''+100y = 2\cos(\omega t)$ has solutions displaying *resonance* when

(a) $\omega = 10,000$
(b) $\omega = 10$
(c) $\omega = 9$
(d) All of the above
(e) None of the above

460. The differential equation $y'' + 100y = 2\cos(\omega t)$ has solutions displaying *beats* when

(a) $\omega = 10,000$
(b) $\omega = 10$
(c) $\omega = 9$
(d) All of the above
(e) None of the above

461. The differential equation $y'' + 4y = 2\cos(2t)$ has solutions clearly displaying

(a) beats
(b) damping
(c) resonance
(d) All of the above
(e) None of the above

462. The differential equation $4y'' + y = 2\cos(4t)$ has solutions clearly displaying

(a) beats
(b) damping
(c) resonance
(d) All of the above

(e) None of the above

463. The differential equation $4y'' + 4y = 2\cos(t)$ has solutions clearly displaying

(a) beats
(b) damping
(c) resonance
(d) All of the above
(e) None of the above

Second Order Differential Equations as Systems

464. A standard approach to converting second order equations such as $x'' = x' - 2x + 4$ is to introduce a new variable, y, such that:

(a) $y' = x$
(b) $y = x'$
(c) $y = x$
(d) $y' = x'$

465. A first-order system equivalent to the second order differential equation $x''+2x'+x = 2$ is:

(a)

$$\begin{aligned} x' &= y \\ y' &= x - 2x' + 2 \end{aligned}$$

(b)

$$\begin{aligned} x' &= y \\ y' &= -2x + y + 2 \end{aligned}$$

(c)

$$\begin{aligned} x' &= y \\ y' &= -x - 2y + 2 \end{aligned}$$

(d)

$$\begin{aligned} x' &= y \\ y' &= -x + 2y + 2 \end{aligned}$$

466. Which second-order differential equation is equivalent to the first-order system below?

$$\begin{aligned} x' &= y \\ y' &= 2x + 4y \end{aligned}$$

(a) $y' = 2x + 4x'$
(b) $x'' - 4x' - 2x = 0$
(c) $x'' + 4x' + 2x = 0$
(d) None of the above

467. Which second-order differential equation is equivalent to the first-order system below?

$$\begin{aligned} x' &= -3x + y \\ y' &= x - 2y \end{aligned}$$

(a) $x'' + 5x' + 5x = 0$
(b) $x' = -3x + x' + 3x$
(c) $y'' + 5y' + 5y = 0$
(d) $y' = -2y - (y' + 2y)$
(e) This system can not be converted to a second-order equation.

468. In the spring mass system described by $x'' = -2x' - 2x$, what does the variable x represent?

(a) The spring's displacement from equilibrium
(b) The mass's displacement from equilibrium
(c) The spring's velocity
(d) The mass's velocity
(e) None of the above

469. The spring mass system described by $x'' = -2x' - 2x$, can be converted to a first-order system by introducing the new variable $y = x'$. What does y represent?

(a) The mass's displacement from equilibrium

(b) The mass's velocity

(c) The mass's acceleration

(d) None of the above

470. A first-order system equivalent to the spring mass system $x'' = -2x' - 2x$ is:

(a)

$$\begin{aligned} x' &= y \\ y' &= -2x - 2x' \end{aligned}$$

(b)

$$\begin{aligned} x' &= y \\ y' &= -2y + 2x \end{aligned}$$

(c)

$$\begin{aligned} x' &= y \\ y' &= -2y - 2x \end{aligned}$$

(d)

$$\begin{aligned} x' &= y \\ y' &= 2y - 2x \end{aligned}$$

471. The solution to the spring mass system $x'' = -2x' - 2x$ is:

(a) $\begin{bmatrix} x \\ y \end{bmatrix} = c_1 e^{-2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^{t} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(b) $\begin{bmatrix} y \\ x \end{bmatrix} = c_1 e^{-2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^{t} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(c) $\begin{bmatrix} x \\ y \end{bmatrix} = e^{-t} \left(c_1 \cos t + c_2 \sin t \right) \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{-t} \left(-c_1 \sin t + c_2 \cos t \right) \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \end{bmatrix} = e^{-t}\left(c_1 \cos t + c_2 \sin t\right)\begin{bmatrix} 1 \\ -2 \end{bmatrix} + e^{-t}\left(-c_1 \sin t + c_2 \cos t\right)\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

472. The position of the mass in the spring mass system $x'' = -2x' - 2x$ is given by:

(a) $y = -2c_1 e^{-t} \cos t - 2c_2 e^{-t} \sin t$
(b) $y = c_1 e^{-t}\left(\cos t - \sin t\right) + c_2 e^{-t}\left(\cos t + \sin t\right)$
(c) $x = c_1 e^{-t}\left(\cos t - \sin t\right) + c_2 e^{-t}\left(\cos t + \sin t\right)$
(d) $x = -2c_1 e^{-t} \cos t - 2c_2 e^{-t} \sin t$

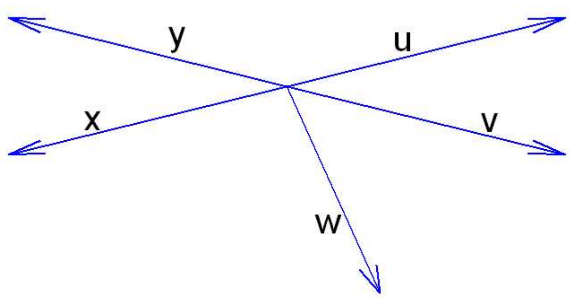
Chapter 5: Linear Transofmration

Linear Transformations and Projections

473. Define $T(v) = Av$, where $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Then $T(v)$

(a) reflects v about the x_2-axis.
(b) reflects v about the x_1-axis.
(c) rotates v clockwise $\pi/2$ radians about the origin.
(d) rotates v counterclockwise $\pi/2$ radians about the origin.
(e) None of the above

474. Define $T(u) = Au$, where $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Using the vectors from $\Re^2$ plotted below, this means that



(a) $T(u) = v$.
(b) $T(u) = w$.
(c) $T(u) = x$.

(d) $T(u) = y$.

(e) None of the above

475. If the linear transformation $T(v) = Av$ rotates the vectors $v_1 = (-1, 0)$ and $v_2 = (0, 1)$ clockwise $\pi/2$ radians, the resulting vectors are

(a) $T(v_1) = \left(-\sqrt{2}/2, \sqrt{2}/2\right)$ and $T(v_2) = \left(\sqrt{2}/2, \sqrt{2}/2\right)$

(b) $T(v_1) = \left(-\sqrt{2}/2, -\sqrt{2}/2\right)$ and $T(v_2) = \left(-\sqrt{2}/2, \sqrt{2}/2\right)$

(c) $T(v_1) = (0, -1)$ and $T(v_2) = (-1, 0)$

(d) $T(v_1) = (0, 1)$ and $T(v_2) = (1, 0)$

(e) None of the above

476. If the linear transformation $T(v) = Av$ rotates the vectors $(-1, 0)$ and $(0, 1)$ clockwise $\pi/2$ radians then

(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(d) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(e) None of the above

477. If the linear transformation $T(v) = Av$ rotates the vectors $v_1 = (-1, 0)$ and $v_2 = (0, 1)$ clockwise π radians, the resulting vectors are

(a) $T(v_1) = (1, 0)$ and $T(v_2) = (0, -1)$

(b) $T(v_1) = (-1, 0)$ and $T(v_2) = (0, 1)$

(c) $T(v_1) = (0, 1)$ and $T(v_2) = (1, 0)$

(d) $T(v_1) = (0, -1)$ and $T(v_2) = (-1, 0)$

(e) None of the above

478. If the linear transformation $T(v) = Av$ rotates the vectors $(-1, 0)$ and $(0, 1)$ π radians clockwise then

(a) $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(d) $A = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$

(e) None of the above

479. If the linear transformation $T(v) = Av$ rotates the vector v θ radians clockwise, then

(a) $A = \begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$

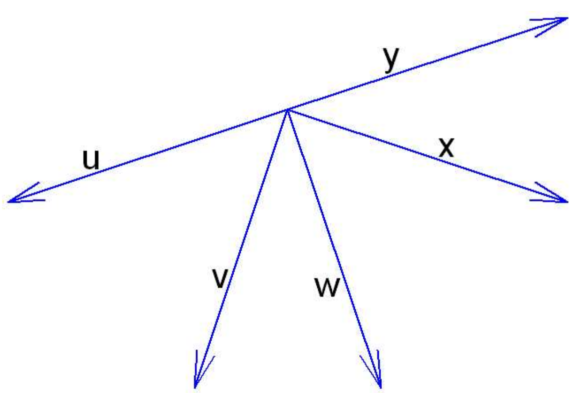
(b) $A = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$

(c) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

(d) $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(e) None of the above

480. The linear transformation $T(v) = Av$ produces $T(u) = w$, $T(v) = x$ and $T(w) = y$, as shown below. Which of the following could be the matrix A?



(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(d) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(e) None of the above

481. The linear transformation $T(x, y) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$, can be written as

(a) $T(x, y) = (x, y)$

(b) $T(x, y) = (y, x)$

(c) $T(x, y) = (-x, y)$

(d) $T(x, y) = (-y, x)$

(e) None of the above

482. The linear transformation $T(x, y) = (x + 2y, x - 2y)$, can be written as a matrix transformation $T(x, y) = A \begin{bmatrix} x \\ y \end{bmatrix}$ where

(a) $A = \begin{bmatrix} x & 2y \\ x & -2y \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$

(d) It can't be written in matrix form

483. Which of the following is not a linear transformation?

(a) $T(x, y) = (x, y + 1)$

(b) $T(x, y) = (x - 2y, x)$

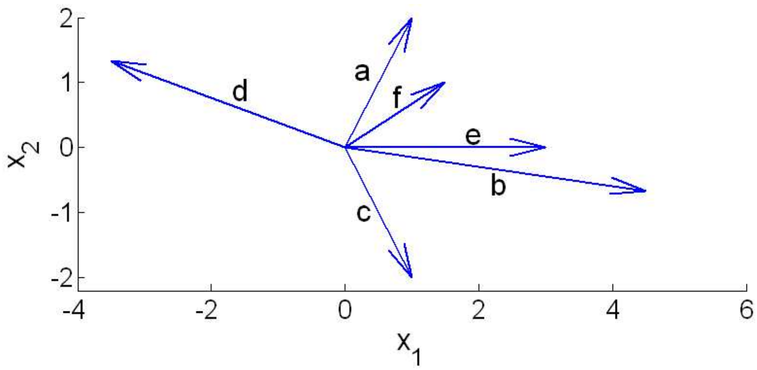
(c) $T(x, y) = (4y, x - 2y)$

(d) $T(x, y) = (x, 0)$

(e) All are linear transformations

(f) More than one of these are not linear transforms

484. **True or False** If a transformation produces $T(a) = b$, $T(c) = d$, and $T(e) = f$, for the vectors plotted below, then this transformation must be nonlinear.



(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

485. Is the transformation $T(x, y, z) = (x, y, 0)$ linear?

(a) No, it is not linear because all z components map to 0.

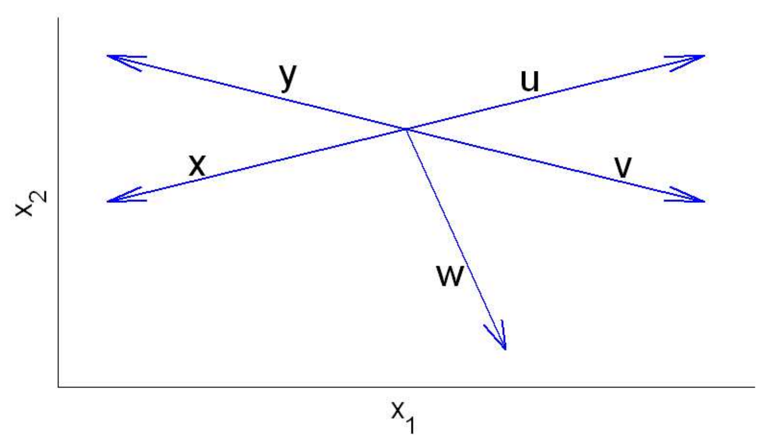
(b) No, it is not linear because it does not satisfy the scalar multiplication property.

(c) No, it is not linear because it does not satisfy the vector addition property.

(d) No, it is not linear for a reason not listed here.

(e) Yes, it is linear.

486. **True or False** If a transformation produces $T(x) = y$, $T(y) = u$, $T(u) = v$, and $T(v) = w$ for the vectors plotted below, then this transformation must be nonlinear.



(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

487. If f is a function, is the transformation $T(f) = f'$ linear?

(a) No, it is not linear because it does not satisfy the scalar multiplication property.

(b) No, it is not linear because it does not satisfy the vector addition property.

(c) No, it is not linear for a reason not listed here.

(d) Yes, it is linear.

488. What is the range of $T(v) = Av$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 0 \end{bmatrix}$?

(a) All of $\Re^3$

(b) All of $\Re^2$

(c) A line in $\Re^2$

(d) A plane in $\Re^3$

(e) A line in $\Re^3$

489. When we map w to Aw and w is an eigenvector of A, what is the geometric effect?

(a) Aw is a rotation of w.

(b) Aw is a reflection of w in the x-axis.

(c) Aw is a reflection of w in the y-axis.

(d) Aw is parallel to w but may have a different length.

Eigenvalues and Eigenvectors

490. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

(e) None of the above

491. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^2 \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(a) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 6 \\ 6 \end{bmatrix}$

(c) $\begin{bmatrix} 9 \\ 9 \end{bmatrix}$

(d) $\begin{bmatrix} 12 \\ 12 \end{bmatrix}$

(e) None of the above

(f) This matrix multiplication is impossible.

492. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^4 \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(a) $\begin{bmatrix} 27 \\ 27 \end{bmatrix}$

(b) $\begin{bmatrix} 81 \\ 81 \end{bmatrix}$

(c) $\begin{bmatrix} 243 \\ 243 \end{bmatrix}$

(d) $\begin{bmatrix} 729 \\ 729 \end{bmatrix}$

(e) None of the above

493. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(a) $\begin{bmatrix} 3n \\ 3n \end{bmatrix}$

(b) $3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c) $n^3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(d) $3^n \begin{bmatrix} n \\ n \end{bmatrix}$

(e) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}^n$

(f) More than one of the above

494. Suppose A is an $n \times n$ matrix, c is a scalar, and x is an $n \times 1$ vector. If $Ax = cx$, what is A^2x?

(a) $2cx$

(b) c^2x

(c) cx

(d) None of the above

495. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(a) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ -3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $\begin{bmatrix} -3 \\ 3 \end{bmatrix}$

(e) None of the above

496. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(a) $(-1)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) $(-1)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(c) $(-3)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $\begin{bmatrix} (-1)^n \\ (-1)^{n+1} \end{bmatrix}$

(e) None of the above

(f) More than one of the above

497. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(a) $3^n \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(b) $2^n \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

(c) $6^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(d) $3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

498. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

(a) $3^n \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

(b) $(-1)^n \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

(c) $(-5)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $5 \begin{bmatrix} (-1)^n \\ (-1)^n \end{bmatrix}$

(e) None of the above

(f) More than one of the above

499. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(a) $\begin{bmatrix} 3 \\ 15 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ -5 \end{bmatrix}$

(c) $\begin{bmatrix} 11 \\ 7 \end{bmatrix}$

(d) $\begin{bmatrix} 7 \\ 11 \end{bmatrix}$

(e) None of the above

500. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(a) $11^n \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(b) $7^n \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(c) $\begin{bmatrix} 11^n \\ 7^n \end{bmatrix}$

(d) $\begin{bmatrix} 25 \\ 29 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

501. Write the vector $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \frac{1}{2}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$

(c) $\begin{bmatrix} 1 \\ 5 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) None of the above

(e) More than one of the above

502. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(a) $-1 \times 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 3 \times (-2)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) $3 \times (-1)^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-2) \times 3^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) $3 \times 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-2) \times (-1)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $3 \times 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-1) \times (-2)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

503. Which of the following is an eigenvector of the matrix $\begin{bmatrix} 2 & -1 \\ -4 & -1 \end{bmatrix}$?

(a) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ -4 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

504. Which of the following is an eigenvector of the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$?

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} -3 \\ -3 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ -2/3 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

505. Suppose the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has an eigenvalue 1 with associated eigenvector $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. What is $A^{50}x$?

(a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(b) $\begin{bmatrix} a^{50} & b^{50} \\ c^{50} & d^{50} \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2^{50} \\ 3^{50} \end{bmatrix}$

(e) Way too hard to compute.

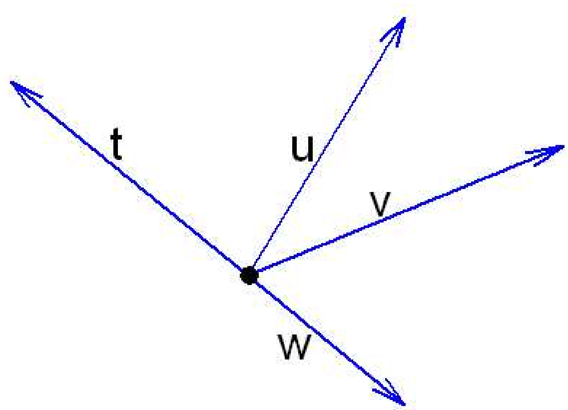
506. Vector x is an eigenvector of matrix A. If $x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $Ax = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$, then what is the associated eigenvalue?

(a) 1
(b) 3
(c) 4
(d) Not enough information is given.

507. Which of the following is an eigenvector of $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$? (You should be able to answer this by checking the vectors given, rather than by finding the eigenvectors of A from scratch.)

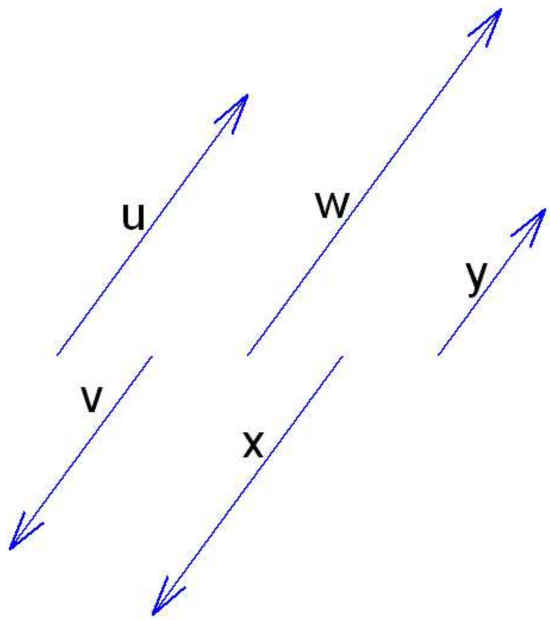
(a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$
(b) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
(d) None of the above

508. The vector t is an eigenvector of the matrix A. What could be the result of the product At?



(a) $At = u$
(b) $At = v$
(c) $At = w$
(d) None of the above

509. The vector u is an eigenvector of the matrix A and $Au = v$, where the vectors u and v are shown below. What could be the result of the product Av?



(a) $Av = u$

(b) $Av = v$

(c) $Av = w$

(d) $Av = x$

(e) $Av = y$

510. $\begin{bmatrix} 4/3 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$. What is the associated eigenvalue? (Think! Don't solve for all the eigenvalues and eigenvectors.)

(a) 4/3

(b) 5

(c) -2

511. The matrix $A = \begin{bmatrix} -1 & 4 \\ 3 & 0 \end{bmatrix}$ has an eigenvalue 3 with associated eigenvector $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Let $y = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Which of the following statements is true?

(a) $Ax = 3x$

(b) $Ay = 3y$

(c) For any scalars c and d, $A(cx + dy) = 3(cx + dy)$

(d) All of the above are true.

(e) Only (a) and (b) are true.

512. The matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ has an eigenvalue 2 with associated eigenvectors $x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Which of the following statements is true?

(a) $Ax = 2x$

(b) $Ay = 2y$

(c) For any scalars c and d, $A(cx + dy) = 2(cx + dy)$.

(d) For any nonzero scalars c and d, $cx + dy$ is an eigenvector of A corresponding to the eigenvalue 2.

(e) All of the above are true.

(f) Only (a) and (b) are true.

513. **True or False** Any nonzero linear combination of two eigenvectors of a matrix A is an eigenvector of A.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

514. If w is an eigenvector of A, how does the vector Aw compare geometrically to the vector w?

(a) Aw is a rotation of w.

(b) Aw is a reflection of w in the x-axis.

(c) Aw is a reflection of w in the y-axis.

(d) Aw is parallel to w but may have a different length.

515. What does it mean if 0 is an eigenvalue of a matrix A?

(a) The determinant of A is zero.

(b) The columns of A are linearly dependent.

(c) There are an infinite number of solutions to the system $Ax = 0$.

(d) All of the above

(e) None of the above

516. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 4 & 2 \end{bmatrix}$ and note that all of the rows sum to six. Which of the following is true?

(a) $w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of A.

(b) 6 is an eigenvalue of A.

(c) Both statements are true.

(d) Neither statement is true.

Eigenspaces

517. If a vector x is in the eigenspace of A corresponding to λ, and $\lambda \neq 0$, then x is

(a) in the nullspace of the matrix A.

(b) in the nullspace of the matrix $A - \lambda I$.

(c) not the zero vector.

(d) More than one of the above correctly completes the sentence.

518. Which of the following statements is correct?

(a) The set of eigenvectors of a matrix A forms the eigenspace of A.

(b) The set of eigenvectors of a matrix A spans the eigenspace of A.

(c) Since any multiple of an eigenvector is also an eigenvector, the eigenspace always has infinite dimension.

(d) More than one of the above statements are correct.

(e) None of the above statements are correct.

519. Which of the following statements is correct?

(a) The set of eigenvectors of a matrix A corresponding to a particular eigenvalue λ_1, together with the zero vector, forms the eigenspace of A corresponding to λ_1.

(b) An eigenspace corresponding to a non-repeated eigenvalue has dimension one.

(c) An eigenvalue of multiplicity two has a corresponding eigenspace of dimension two.

(d) All of the above statements are correct.

(e) Exactly two of the above statements are correct.

Complex Eigenvalues

520. **True or False** Real matrices have only real eigenvalues.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

521. Which of the following could *not* be the set of distinct eigenvalues for a 3×3 real matrix?

(a) 2, 5

(b) 1, 3, 5

(c) 2, 3, $4 + 7i$

(d) 3, $2 + i$, $2 - i$

522. **True or False** Real eigenvalues of a real matrix correspond to real eigenvectors only.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

Diagonalization

523. What are the eigenvalues of $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$?

(a) 2 and 3
(b) 0 and 2
(c) 0 and 3
(d) 5 and 6

524. If $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, what is D^5?

(a) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
(b) $\begin{bmatrix} 10 & 0 \\ 0 & 15 \end{bmatrix}$
(c) $\begin{bmatrix} 2^5 & 0 \\ 0 & 3^5 \end{bmatrix}$
(d) Too hard to compute by hand.

525. Why might we be interested in diagonalizing a matrix?

(a) Because it is easy to find the eigenvalues of a diagonal matrix.
(b) Because it is easy to compute powers of a diagonal matrix.
(c) Both of these reasons.

526. Which of the following statements are true?

(a) An $n \times n$ matrix with n linearly independent eigenvectors is diagonalizable.
(b) Any diagonalizable $n \times n$ matrix has n linearly independent eigenvectors.
(c) Both are true.
(d) Neither is true.

527. Which of the following statements are true?

(a) An $n \times n$ matrix with n distinct eigenvalues is diagonalizable.
(b) Any diagonalizable $n \times n$ matrix has n distinct eigenvalues.
(c) Both are true.
(d) Neither is true.

528. Which of the following statements are true?

(a) If A is a diagonalizable matrix, then A does not have any zero eigenvalues.
(b) If A does not have any zero eigenvalues, then A is diagonalizable.
(c) Both are true.
(d) Neither is true.

529. **True or False** Invertible matrices are diagonalizable.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

Chapter 6: Linear Systems of Differential Equations

Testing Solutions to Linear Systems

530. We want to test the solution $x_1 = -e^{-2t}$ and $x_2 = e^{-2t}$ in the system

$$\begin{aligned} x_1' &= x_1 + 3x_2 \\ x_2' &= 3x_1 + x_2 \end{aligned}$$

What equations result from substituting the solution into the equation?

(a)

$$\begin{aligned} -e^{-2t} &= -e^{-2t} + 3e^{-2t} \\ e^{-2t} &= -3e^{-2t} + e^{-2t} \end{aligned}$$

(b)

$$\begin{aligned} -e^{-2t} &= e^{-2t} - 3e^{-2t} \\ e^{-2t} &= 3e^{-2t} - e^{-2t} \end{aligned}$$

(c)

$$\begin{aligned} 2e^{-2t} &= -e^{-2t} + 3e^{-2t} \\ -2e^{-2t} &= -3e^{-2t} + e^{-2t} \end{aligned}$$

(d)

$$\begin{aligned} -2e^{-2t} &= e^{-2t} - 3e^{-2t} \\ 2e^{-2t} &= 3e^{-2t} - e^{-2t} \end{aligned}$$

(e) None of the above

531. Is $x_1 = x_2 = x_3 = e^t$ a solution to the system below?

$$\begin{aligned} x_1' &= 3x_1 - x_2 + x_3 \\ x_2' &= 2x_1 - x_3 \\ x_3' &= x_1 - x_2 + x_3 \end{aligned}$$

(a) Yes, it is a solution.
(b) No, it is not a solution because it does not satisfy $x_1' = 3x_1 - x_2 + x_3$.
(c) No, it is not a solution because it does not satisfy $x_2' = 2x_1 - x_3$.
(d) No, it is not a solution because it does not satisfy $x_3' = x_1 - x_2 + x_3$.
(e) No, it is not a solution because it doesn't work in any equation for all values of t.

532. Which of the following are solutions to the system below?

$$\begin{aligned} x_1' &= 4x_1 - x_2 \\ x_2' &= 2x_1 + x_2 \end{aligned}$$

(a)

$$\begin{aligned} x_1 &= e^{2t} \\ x_2 &= e^{2t} \end{aligned}$$

(b)

$$\begin{aligned} x_1 &= e^{2t} \\ x_2 &= 2e^{2t} \end{aligned}$$

(c)

$$\begin{aligned} x_1 &= e^{3t} \\ x_2 &= e^{-3t} \end{aligned}$$

(d) None of the above.

(e) All of the above.

533. Since we know that both $x_1 = x_2 = e^{3t}$ and $x_1 = e^{-t}, x_2 = -e^{-t}$ are solutions to the system

$$\begin{aligned} x_1' &= x_1 + 2x_2 \\ x_2' &= 2x_1 + x_2 \end{aligned}$$

Which of the following are also solutions?

(a)

$$\begin{aligned} x_1 &= 3e^{3t} - e^{-t} \\ x_2 &= 3e^{3t} + e^{-t} \end{aligned}$$

(b)

$$\begin{aligned} x_1 &= -e^{3t} - e^{-t} \\ x_2 &= -e^{3t} + e^{-t} \end{aligned}$$

(c)

$$\begin{aligned} x_1 &= 2e^{3t} + 4e^{-t} \\ x_2 &= -4e^{-t} + 2e^{3t} \end{aligned}$$

(d)

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 0 \end{aligned}$$

(e) None of the above.

(f) All of the above.

534. Consider the system of differential equations,

$$y'(t) = \begin{bmatrix} 14 & 0 & -4 \\ 2 & 13 & -8 \\ -3 & 0 & 25 \end{bmatrix} y(t)$$

.

Which of the following functions solve this system?

(a) $y(t) = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} e^{-4t}$

(b) $y(t) = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} e^{6t}$

(c) $y(t) = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} e^{13t}$

(d) None of the above.

(e) All of the above.

Solutions to Linear Systems

535. Consider the linear system given by

$$\frac{d\vec{Y}}{dt} = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix} \vec{Y}.$$

True or False: $\vec{Y}_1(t) = \begin{pmatrix} e^{2t} \\ 0 \end{pmatrix}$ is a solution.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

536. Consider the linear system $\dfrac{d\vec{Y}}{dt} = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix} \vec{Y}$ with solution $\vec{Y}_1(t) = \begin{pmatrix} e^{2t} \\ 0 \end{pmatrix}$.

True or False: The function $k \cdot \vec{Y}_1(t)$ formed by multiplying $\vec{Y}_1(t)$ by a constant k is also a solution to the linear system.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

537. Consider the linear system $\dfrac{d\vec{Y}}{dt} = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix} \vec{Y}$. The functions $\vec{Y}_1(t)$ and $\vec{Y}_2(t)$ are solutions to the linear system.

True or False: The function $\vec{Y}_1(t) + \vec{Y}_2(t)$ formed by adding the two solutions $\vec{Y}_1(t)$ and $\vec{Y}_2(t)$ is also a solution to the linear system.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

538. **True or False:** The functions $\vec{Y}_1(t) = \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix}$ and $\vec{Y}_2(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$ are linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

539. **True or False:** The functions $\vec{Y}_1(t) = \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix}$ and $\vec{Y}_2(t) = \begin{pmatrix} -2\sin(t) \\ -2\cos(t) \end{pmatrix}$ are linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

540. If we are told that the general solution to the linear homogeneous system $Y' = AY$ is $Y = c_1 e^{-4t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, then an equivalent form of the solution is

(a) $y_1 = -2c_1 e^{-4t} + c_2 e^{-4t}$ and $y_2 = 2c_1 e^{3t} + 3c_2 e^{3t}$
(b) $y_1 = -2c_1 e^{-4t} + 2c_2 e^{3t}$ and $y_2 = c_1 e^{-4t} + 3c_2 e^{3t}$
(c) $y_1 = -2c_1 e^{-4t} + c_1 e^{-4t}$ and $y_2 = 2c_2 e^{3t} + 3c_2 e^{3t}$
(d) $y_1 = -2c_1 e^{-4t} + 2c_1 e^{3t}$ and $y_2 = c_2 e^{-4t} + 3c_2 e^{3t}$
(e) All of the above
(f) None of the above

541. If $Y = e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + e^{-4t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is a solution to the linear homogeneous system $Y' = AY$, which of the following is also a solution?

(a) $Y = 2e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
(b) $Y = 3e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} - 4e^{-4t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$
(c) $Y = 1/4e^{-4t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$
(d) All of the above
(e) None of the above

542. The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = 4$ with $v_1 =< 1, 2 >$ and $\lambda_2 = -3$ with $v_2 =< -2, 1 >$. What is a form of the solution?

(a) $Y = c_1 e^{-4t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(b) $Y = c_1 e^{4t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(c) $Y = c_1 e^{4t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(d) $Y = c_1 e^{-4t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

543. You have a linear, homogeneous, system of differential equations with constant coefficients whose coefficient matrix A has the eigensystem: eigenvalues -5 and -2 and eigenvectors $< -1, 2 >$ and $< -4, 5 >$, respectively. Then a general solution to $\dfrac{d\vec{Y}}{dt} = A\vec{Y}$ is given by:

(a) $Y = \begin{bmatrix} -k_1 e^{-5t} + 2k_2 e^{-2t} \\ -4k_1 e^{-5t} + 5k_2 e^{-2t} \end{bmatrix}$

(b) $Y = \begin{bmatrix} -k_1 e^{-2t} - 4k_2 e^{-5t} \\ 2k_1 e^{-2t} + 5k_2 e^{-5t} \end{bmatrix}$

(c) $Y = \begin{bmatrix} -k_1 e^{-5t} - 4k_2 e^{-2t} \\ 2k_1 e^{-5t} + 5k_2 e^{-2t} \end{bmatrix}$

(d) $Y = \begin{bmatrix} -k_1 e^{-2t} + 2k_2 e^{-5t} \\ -4k_1 e^{-2t} + 5k_2 e^{-5t} \end{bmatrix}$

544. The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = 4$ with $v_1 =< 1, 2 >$ and $\lambda_2 = -3$ with $v_2 =< -2, 1 >$. In the long term, phase trajectories:

(a) become parallel to the vector $v_2 =< -2, 1 >$.

(b) tend towards positive infinity.

(c) become parallel to the vector $v_1 =< 1, 2 >$.

(d) tend towards 0.

(e) None of the above

545. If the eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = -4$ with $v_1 =< 1, 2 >$ and $\lambda_2 = 3$ with $v_2 =< 2, 3 >$, is $y_a = e^{-4t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ a solution?

(a) Yes, it is a solution.

(b) No, it is not a solution because it does not contain λ_2.

(c) No, it is not a solution because it is a vector.

(d) No, it is not a solution because of a different reason.

546. The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = 4$ with $v_1 =< 1, 0 >$ and $\lambda_2 = 4$ with $v_2 =< 0, 1 >$. What is a form of the solution?

(a) $Y = c_1 e^{4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(b) $Y = c_1 e^{-4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(c) $Y = c_1 e^{4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 t e^{4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(d) $Y = c_1 e^{-4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

547. The system of differential equations $Y' = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} Y$ has eigenvalue $\lambda = 2$ with multiplicity 2, and all eigenvectors are multiples of $v =< 1, -1 >$. Testing $Y = e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, we find that

(a) Y is a solution.

(b) Y is not a solution.

548. The system of differential equations $Y' = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} Y$ has eigenvalue $\lambda = 2$ with multiplicity 2, and all eigenvectors are multiples of $v =< 1, -1 >$. One solution to this equation is $Y = e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Testing $Y = te^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, we find that

(a) $Y = te^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is also a solution.

(b) $Y = te^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is not a solution.

549. The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda = -4$ with multiplicity 2, and all eigenvectors are multiples of $v =< 1, -2 >$. What is the form of the general solution?

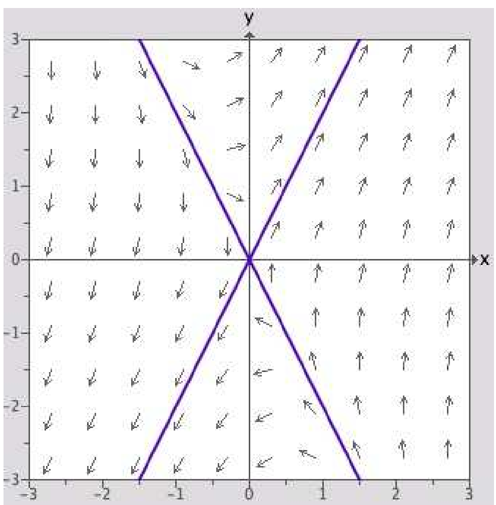
(a) $Y = c_1 e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 t e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(b) $Y = c_1 e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(c) $Y = c_1 e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 \left(t e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + e^{-4t} \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right)$

Geometry of Systems

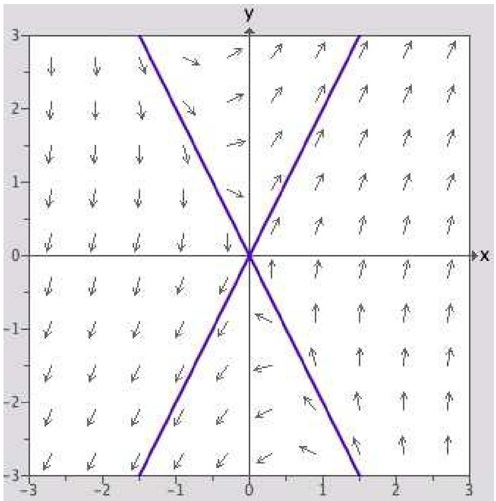
550. The differential equation $\frac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below. We denote the associated eigenvalues by λ_1 and λ_2.



We can deduce that λ_1 is

(a) positive real
(b) negative real
(c) zero
(d) complex
(e) There is not enough information

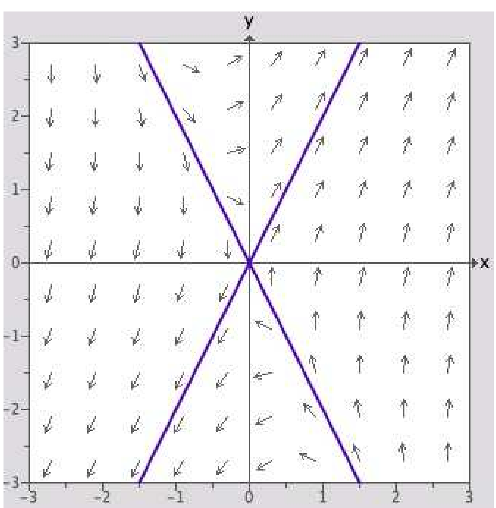
551. The differential equation $\frac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below. We denote the associated eigenvalues by λ_1 and λ_2.



We can deduce that λ_2 is

(a) positive real
(b) negative real
(c) zero
(d) complex
(e) There is not enough information

552. The differential equation $\dfrac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below.



Suppose we have a solution $\vec{Y}(t)$ to this system of differential equations which satisfies initial condition $\vec{Y}(t) = (x_0, y_0)$ where the point (x_0, y_0) is not on the line through the point $(1, -2)$. Which statement best describes the behavior of the solution as $t \to \infty$?

(a) The solution tends towards the origin.

(b) The solution moves away from the origin and asymptotically approaches the line through $< 1, 2 >$.

(c) The solution moves away from the origin and asymptotically approaches the line through $< 1, -2 >$.

(d) The solution spirals and returns to the point (x_0, y_0).

(e) There is not enough information.

553. Suppose you have a linear, homogeneous system of differential equations with constant coefficients whose coefficient matrix has the following eigensystem: eigenvalues -4 and -1 and eigenvectors $< 1, 1 >$ and $< -2, 1 >$ respectively. The function $\vec{Y}(t)$ is a solution to this system of differential equations which satisfies initial value $\vec{Y}(0) = (-15, 20)$. Which statement best describes the behavior of the solution as $t \to \infty$?

(a) The solution tends towards the origin.

(b) The solution moves away from the origin and asymptotically approaches the line through $< 1, 1 >$.

(c) The solution moves away from the origin and asymptotically approaches the line through $< -2, 1 >$.

(d) The solution spirals and returns to the point $(-15, 20)$.

(e) There is not enough information.

554. Suppose we have a linear, homogeneous system of differential equations with constant coefficients who coefficient matrix has the following eigensystem: eigenvalues -4 and -1 and eigenvectors $< 1, 1 >$ and $< -2, 1 >$ respectively. Suppose we have a solution $\vec{Y}(t)$ which satisfies $\vec{Y}(0) = (x_0, y_0)$ where the point (x_0, y_0) is not on the line through the point $(1, 1)$. How can we best describe the manner in which the solution $\vec{Y}(t)$ approaches the origin?

(a) The solution will approach the origin in the same manner as the line which goes through the point $(1, 1)$.

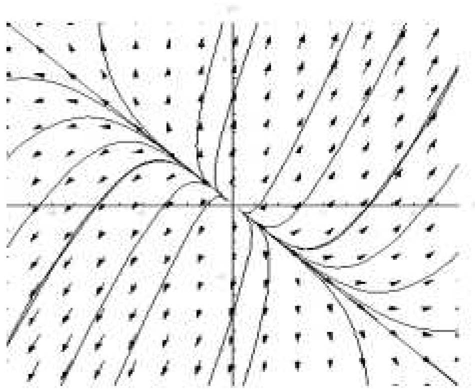
(b) The solution will approach the origin in the same manner as the line which goes through the point $(-2, 1)$.

(c) The solution will directly approach the origin in a straight line from the point (x_0, y_0).

(d) The answer can vary greatly depending on what the point (x_0, y_0) is.

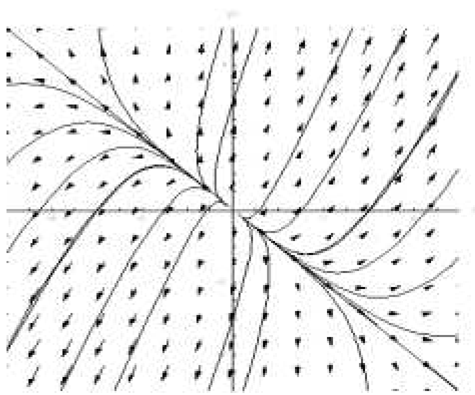
(e) The solution doesn't approach the origin.

555. Using the phase portrait below for the system $Y' = AY$, we can deduce that the eigenvalues of the coefficient matrix A are:



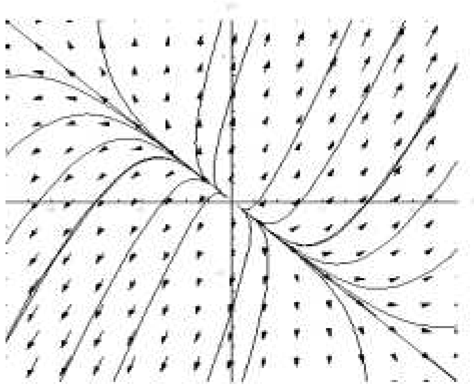
(a) both real
(b) both complex
(c) one real, one complex
(d) Not enough information is given

556. Using the phase portrait below for the system $Y' = AY$, we can deduce that the eigenvalues are:



(a) of mixed sign
(b) both negative
(c) both positive
(d) Not enough information is given

557. Using the phase portrait below for the system $Y' = AY$, we can deduce that the dominant eigenvector is:



(a) $< -1, 1 >$

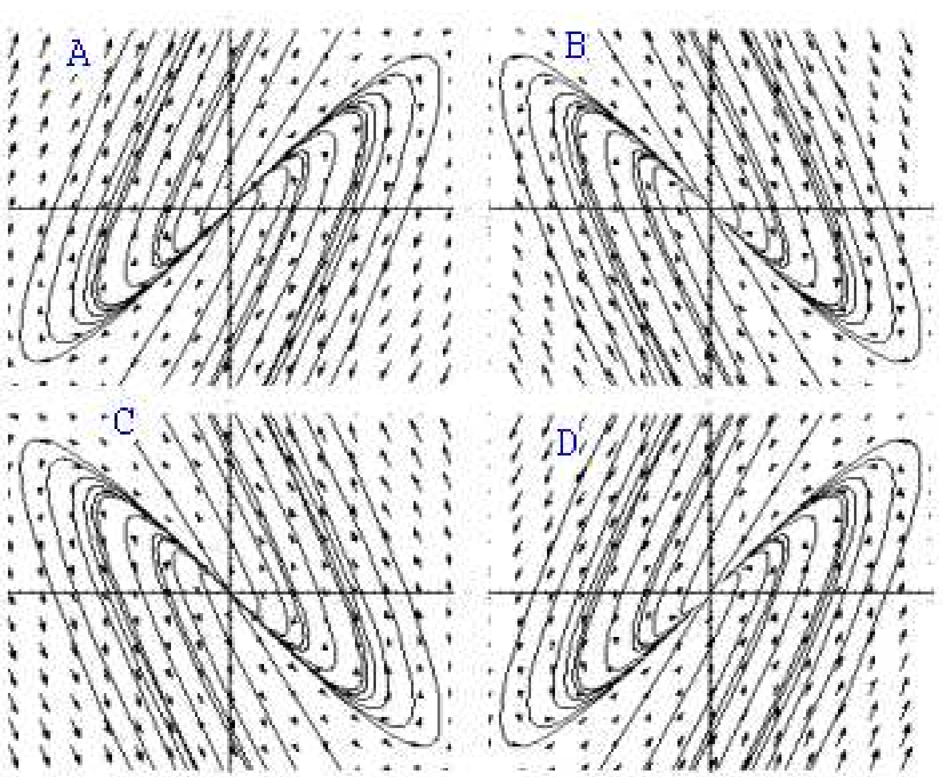
(b) $< 1, 3 >$

(c) $< 1, -2 >$

(d) There is no dominant eigenvector because there is no vector that is being approached by all of the solution curves.

(e) Not enough information is given

558. Which phase portrait below corresponds to the linear, homogeneous, system of differential equations with constant coefficients whose coefficient matrix has the following eigensystem: eigenvalues -5 and -2 and eigenvectors $< -1, 2 >$ and $< -4, 5 >$, respectively?



559. Classify the equilibrium point at the origin for the system

$$\frac{d\vec{Y}}{dt} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \vec{Y}.$$

(a) Sink
(b) Source
(c) Saddle
(d) None of the above

Nonhomogeneous Linear Systems

560. Which of the following would be an appropriate guess for the particular solution to the forced ODE $y' = -3y + t^2$?

(a) $y_p = c_1 t^2$
(b) $y_p = c_1 + c_2 t + c_3 t^2$
(c) $y_p = c_1 e^{-3t} + c_2 t^2$
(d) $y_p = c_1 e^{-3t} + c_2 t^2 + c_3 t + c_4$
(e) $y_p = c_1 + c_2 t^2$

561. Which of the following would be an appropriate guess for the particular solution for the decoupled system $Y' = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} Y + \begin{bmatrix} 2e^{4t} \\ e^t \end{bmatrix}$?

(a) $\begin{bmatrix} c_1 e^{4t} + c_2 e^t \\ c_3 e^{4t} + c_4 e^t \end{bmatrix}$
(b) $\begin{bmatrix} c_1 e^t \\ c_2 e^{4t} \end{bmatrix}$
(c) $\begin{bmatrix} c_1 e^{4t} \\ c_2 e^t \end{bmatrix}$
(d) $\begin{bmatrix} 2c_1 e^{4t} \\ c_2 e^t \end{bmatrix}$

562. Which of the following would be an appropriate guess for the particular solution for the system $Y' = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} Y + \begin{bmatrix} e^{4t} \\ e^t \end{bmatrix}$?

(a) $\begin{bmatrix} c_1e^{4t} + c_2e^t \\ c_3e^{4t} + c_4e^t \end{bmatrix}$

(b) $\begin{bmatrix} c_1e^{4t} + c_2e^t \\ c_3e^t \end{bmatrix}$

(c) $\begin{bmatrix} c_1e^{4t} \\ c_2e^t \end{bmatrix}$

(d) $\begin{bmatrix} c_1e^{4t} \\ c_2e^{4t} + c_3e^t \end{bmatrix}$

563. Which of the following is a particular solution for the system $Y' = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} Y + \begin{bmatrix} 6e^{4t} \\ 2e^t \end{bmatrix}$?

(a) $Y_p = \begin{bmatrix} c_1e^{4t} + c_2e^t \\ c_3e^{4t} + c_4e^t \end{bmatrix}$

(b) $Y_p = \begin{bmatrix} 2.4e^{4t} + 0.2e^t \\ 1.2e^{4t} + 0.6e^t \end{bmatrix}$

(c) $Y_p = \begin{bmatrix} 1.05e^{4t} + 0.2e^t \\ 0.3e^{4t} + 0.6e^t \end{bmatrix}$

(d) More than one of the above

564. Which of the following is the general solution for the system $Y' = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} Y + \begin{bmatrix} 6e^{4t} \\ 2e^t \end{bmatrix}$?

(a) $Y = k_1e^{-4t} \begin{bmatrix} -1 \\ 2 \end{bmatrix} + k_2e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1.05e^{4t} + 0.2e^t \\ 0.3e^{4t} + 0.6e^t \end{bmatrix}$

(b) $Y = k_1e^{-4t} + k_2e^{-t} + \begin{bmatrix} 1.05e^{4t} + 0.2e^t \\ 0.3e^{4t} + 0.6e^t \end{bmatrix}$

(c) $Y = k_1e^{4t} \begin{bmatrix} -1 \\ 2 \end{bmatrix} + k_2e^{t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1.05e^{4t} + 0.2e^t \\ 0.3e^{4t} + 0.6e^t \end{bmatrix}$

(d) $Y = k_1e^{-4t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + k_2e^{-t} \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1.05e^{4t} + 0.2e^t \\ 0.3e^{4t} + 0.6e^t \end{bmatrix}$

565. Which of the following would be an appropriate guess for the particular solution for the system $Y' = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} Y + \begin{bmatrix} e^{-4t} \\ e^{-t} \end{bmatrix}$?

(a) $\begin{bmatrix} c_1e^{-4t} + c_2e^{-t} \\ c_3e^{-4t} + c_4e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} c_1e^{-4t} + c_2e^{-t} \\ c_3e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} c_1e^{-4t} \\ c_2e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} c_1e^{-4t} \\ c_2e^{-4t} + c_3e^{-t} \end{bmatrix}$

(e) None of the above

566. Which of the following would be an appropriate guess for the particular solution for the system $Y' = \begin{bmatrix} -2 & 0 \\ 2 & -3 \end{bmatrix} Y + \begin{bmatrix} e^{4t} \\ e^{t} \end{bmatrix}$?

(a) $\begin{bmatrix} c_1e^{4t} + c_2e^{t} \\ c_3e^{4t} + c_4e^{t} \end{bmatrix}$

(b) $\begin{bmatrix} c_1e^{t} \\ c_2e^{4t} + c_3e^{t} \end{bmatrix}$

(c) $\begin{bmatrix} c_1e^{4t} \\ c_2e^{t} \end{bmatrix}$

(d) $\begin{bmatrix} c_1e^{4t} \\ c_2e^{4t} + c_3e^{t} \end{bmatrix}$

567. Which of the following would be an appropriate guess for the particular solution for the system $Y' = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} Y + \begin{bmatrix} \sin \pi t \\ 3t \end{bmatrix}$?

(a) $\begin{bmatrix} c_1 \sin \pi t + c_2 t \\ c_3 \sin \pi t + c_4 t \end{bmatrix}$

(b) $\begin{bmatrix} c_1 \sin \pi t + c_2 \cos \pi t + c_3 t + c_4 \\ c_5 \sin \pi t + c_6 \cos \pi t + c_7 t + c_8 \end{bmatrix}$

(c) $\begin{bmatrix} c_1 \sin \pi t + c_2 t + c_3 \\ c_4 \sin \pi t + c_5 t + c_6 \end{bmatrix}$

(d) $\begin{bmatrix} c_1 \sin \pi t \\ c_2 t + c_3 \end{bmatrix}$

Chapter 7: Nonlinear Systems of Differential Equations

Nonlinear Systems

568. The nonlinear system of differential equations given below has an equilibrium point at $(0, 0)$. Identify the system which represents a linear approximation of the nonlinear system around this point.

$$\frac{dx}{dt} = y + x^2$$
$$\frac{dy}{dt} = -2y + \sin x$$

(a)

$$\frac{dx}{dt} = y + 2x$$
$$\frac{dy}{dt} = -2y$$

(b)

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -2y$$

(c)

$$\frac{dx}{dt} = y + 2x$$
$$\frac{dy}{dt} = -2y + x$$

(d)

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -2y + x$$

569. For the nonlinear system given below, compute the Jacobian $J(x,y)$ that we associate to it.

$$\frac{dx}{dt} = x + 2xy$$
$$\frac{dy}{dt} = -2y + x^2$$

(a)
$$J(x,y) = \begin{pmatrix} 1+2x & 2y \\ 2x & -2 \end{pmatrix}$$

(b)
$$J(x,y) = \begin{pmatrix} 1+2y & 2x \\ 2x & -2 \end{pmatrix}$$

(c)
$$J(x,y) = \begin{pmatrix} 2x & 1+2y \\ -2 & 2x \end{pmatrix}$$

(d)
$$J(x,y) = \begin{pmatrix} 2x & -2 \\ 1+2y & 2x \end{pmatrix}$$

570. The nonlinear system given below has an equilibrium point at $(0,0)$. Classify this point.

$$\frac{dx}{dt} = x + 2xy$$
$$\frac{dy}{dt} = -2y + x^2$$

(a) Sink
(b) Source
(c) Saddle
(d) Spiral Sink
(e) Spiral Source
(f) Center

Euler's Method and Systems of Equations

571. We have the system of differential equations $x' = 3x - 2y$ and $y' = 4y^2 - 7x$. If we know that $x(0) = 2$ and $y(0) = 1$, estimate the values of x and y at $t = 0.1$.

(a) $x(0.1) = 4$, $y(0.1) = -10$
(b) $x(0.1) = 6$, $y(0.1) = -9$
(c) $x(0.1) = 2.4$, $y(0.1) = 0$
(d) $x(0.1) = 0.4$, $y(0.1) = -1$
(e) None of the above

572. We have the system of differential equations $x' = x(-x-2y+5)$ and $y' = y(-x-y+10)$. If we know that $x(4.5) = 3$ and $y(4.5) = 2$, estimate the values of x and y at $t = 4$.

(a) $x(4) = 0$, $y(4) = -3$
(b) $x(4) = 6$, $y(4) = 10$
(c) $x(4) = 6$, $y(4) = 7$
(d) None of the above

573. We have a system of differential equations for $\frac{dx}{dt}$ and $\frac{dy}{dt}$, along with the initial conditions that $x(0) = 5$ and $y(0) = 7$. We want to know the value of these functions when $t = 5$. Using Euler's method and $\Delta t = 1$ we get the result that $x(5) \approx 14.2$ and $y(5) \approx 23.8$. Next, we use Euler's method again with $\Delta t = 0.5$ and find that $x(5) \approx 14.6$ and $y(5) \approx 5.3$. Finally we use $\Delta t = 0.25$, finding that $x(5) \approx 14.8$ and $y(5) \approx -3.7$. What does this mean?

(a) Fewer steps means fewer opportunities for error, so $(x(5), y(5)) \approx (14.2, 23.8)$.
(b) Smaller stepsize means smaller errors, so $(x(5), y(5)) \approx (14.8, -3.7)$.
(c) We have no way of knowing whether any of these estimates is anywhere close to the true values of $(x(5), y(5))$.
(d) At these step sizes we can conclude that $x(5) \approx 15$, but we can only conclude that $y(5) < -3.7$.
(e) Results like this are impossible: We must have made an error in our calculations.

Chapter 8: Laplace Transforms

Laplace Tranforms

574. **True or False** The Laplace transform method is the only way to solve some types of differential equations.

(a) True, and I am very confident

(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

575. Which of the following differential equations would be impossible to solve using the Laplace transform?

(a) $\frac{dc}{dr} = 12c + 3\sin(2r) + 8\cos^2(3r+2)$
(b) $\frac{d^2f}{dx^2} - 100\frac{df}{dx} = \frac{18}{ln(x)}$
(c) $g'(b) = \frac{12}{g}$
(d) $p''(q) = \frac{4}{q} + 96p' - 12p$

576. Suppose we know that zero is an equilibrium value for a certain homogeneous linear differential equation with constant coefficients. Further, suppose that if we begin at equilibrium and we add the nonhomogenous driving term $f(t)$ to the equation, then the solution will be the function $y(t)$. **True or False**: If instead we add the nonhomogeous driving term $2f(t)$, then the solution will be the function $2y(t)$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

577. Suppose we know that zero is an equilibrium value for a certain homogeneous linear differential equation with constant coefficients. Further, suppose that if we begin at equilibrium and we add a nonhomogenous driving term $f(t)$ to the equation that acts only for an instant, at $t = t_1$, then the solution will be the function $y(t)$. **True or False**: Even if we know nothing about the differential equation itself, using our knowledge of $f(t)$ and $y(t)$ we can infer how the system must behave for any other nonhomogeneous driving function $g(t)$ and for any other initial condition.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

578. The Laplace transform of a function $y(t)$ is defined to be a function $Y(s)$ so that $\mathcal{L}[y(t)] = Y(s) = \int_0^\infty y(t)e^{-st}dt$. If we have the function $y(t) = e^{5t}$, then what is its Laplace transform $\mathcal{L}[y(t)] = Y(s)$?

(a) $\mathcal{L}[y(t)] = \frac{1}{s-5}$ if $s > 5$.
(b) $\mathcal{L}[y(t)] = \frac{1}{s-5}$ if $s < 5$.
(c) $\mathcal{L}[y(t)] = \frac{1}{5-s}$ if $s > 5$.
(d) $\mathcal{L}[y(t)] = \frac{1}{5-s}$ if $s < 5$.

579. What is the Laplace transform of the function $y(t) = 1$?

(a) $\mathcal{L}[y(t)] = -\frac{1}{s}$ if $s > 0$.
(b) $\mathcal{L}[y(t)] = -\frac{1}{s}$ if $s < 0$.
(c) $\mathcal{L}[y(t)] = \frac{1}{s}$ if $s > 0$.
(d) $\mathcal{L}[y(t)] = \frac{1}{s}$ if $s < 0$.
(e) This function has no Laplace transform.

580. What is $\mathcal{L}[e^{3t}]$?

(a) $\mathcal{L}[e^{3t}] = \frac{1}{3-s}$ if $s < 3$.
(b) $\mathcal{L}[e^{3t}] = \frac{1}{s-3}$ if $s > 3$.
(c) $\mathcal{L}[e^{3t}] = \frac{1}{s+3}$ if $s > 3$.
(d) $\mathcal{L}[e^{3t}] = \frac{-1}{s+3}$ if $s < 3$.

581. Suppose we have the Laplace transform $Y(s) = \frac{1}{s-2}$ and we want the original function $y(t)$. What is $\mathcal{L}^{-1}\left[\frac{1}{s-2}\right]$?

(a) $\mathcal{L}^{-1}\left[\frac{1}{s-2}\right] = e^{-2t}$
(b) $\mathcal{L}^{-1}\left[\frac{1}{s-2}\right] = e^{2t}$
(c) This cannot be done

582. Suppose we know that the Laplace transform of a particular function $y(t)$ is the function $Y(s)$ so that $\mathcal{L}[y(t)] = Y(s)$. Now, suppose we multiply this function by 5 and take the Laplace transform. What can we say about $\mathcal{L}[5y(t)]$?

(a) $\mathcal{L}[5y(t)] = 5Y(s)$.

(b) $\mathcal{L}[5y(t)] = -5Y(s)$.

(c) $\mathcal{L}[5y(t)] = -\frac{1}{5}Y(s)$.

(d) $\mathcal{L}[5y(t)] = \frac{1}{5}e^{-5}Y(s)$.

(e) None of the above

583. Suppose we know that the Laplace transform of a particular function $f(t)$ is the function $W(s)$ and that the Laplace transform of another function $g(t)$ is the function $X(s)$ so that $\mathcal{L}[f(t)] = W(s)$ and $\mathcal{L}[g(t)] = X(s)$. Now, suppose we multiply these two functions together and take the Laplace transform. What can we say about $\mathcal{L}[f(t)g(t)]$?

(a) $\mathcal{L}[f(t)g(t)] = W(s)X(s)$.

(b) $\mathcal{L}[f(t)g(t)] = \frac{W(s)}{X(s)}$.

(c) $\mathcal{L}[f(t)g(t)] = W(s) + X(s)$.

(d) $\mathcal{L}[f(t)g(t)] = W(s) - X(s)$.

(e) None of the above

584. Suppose we know that the Laplace transform of a particular function $f(t)$ is the function $W(s)$ and that the Laplace transform of another function $g(t)$ is the function $X(s)$ so that $\mathcal{L}[f(t)] = W(s)$ and $\mathcal{L}[g(t)] = X(s)$. Now, suppose we add these two functions together and take the Laplace transform. What can we say about $\mathcal{L}[f(t) + g(t)]$?

(a) $\mathcal{L}[f(t) + g(t)] = W(s)X(s)$.

(b) $\mathcal{L}[f(t) + g(t)] = \frac{W(s)}{X(s)}$.

(c) $\mathcal{L}[f(t) + g(t)] = W(s) + X(s)$.

(d) $\mathcal{L}[f(t) + g(t)] = W(s) - X(s)$.

(e) None of the above

585. Suppose we have $Y(s) = \frac{1}{s-2} + \frac{5}{s+6}$ and we want the function $y(t)$. What is $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right]$?

(a) $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right] = e^{2t} + e^{6t/5}$

(b) $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right] = e^{2t} + 5e^{-6t}$

(c) $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right] = 5e^{2t}e^{-6t}$

(d) $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right] = \frac{e^{2t}e^{6t}}{5}$

(e) This cannot be done

586. Suppose we know that $\mathcal{L}[y(t)] = Y(s)$ and we want to take the Laplace transform of the derivative of $y(t)$, $\mathcal{L}\left[\frac{dy}{dt}\right]$. To get started, recall the method of integration by parts, which tells us that $\int u\, dv = uv - \int v\, du$.

(a) $\mathcal{L}\left[\frac{dy}{dt}\right] = y(t)e^{-st} + sY(s)$.

(b) $\mathcal{L}\left[\frac{dy}{dt}\right] = y(t)e^{-st} - sY(s)$.

(c) $\mathcal{L}\left[\frac{dy}{dt}\right] = y(0) + sY(s)$.

(d) $\mathcal{L}\left[\frac{dy}{dt}\right] = y(0) - sY(s)$.

(e) $\mathcal{L}\left[\frac{dy}{dt}\right] = -y(0) + sY(s)$.

587. We know that $\mathcal{L}[y(t)] = Y(s) = \int_0^\infty y(t)e^{-st}dt$ and that $\mathcal{L}\left[\frac{dy}{dt}\right] = -y(0) + sY(s)$. What do we get when we take the Laplace transform of the differential equation $\frac{dy}{dt} = 2y + 3e^{-t}$ with $y(0) = 2$?

(a) $2 + sY(s) = 2Y(s) + \frac{3}{s+1}$

(b) $2 + sY(s) = \frac{1}{2}Y(s) + \frac{3}{s-1}$

(c) $-2 + sY(s) = 2Y(s) + \frac{3}{s+1}$

(d) $-2 + sY(s) = 2Y(s) - \frac{3}{s+1}$

(e) $2 - sY(s) = \frac{1}{2}Y(s) + \frac{3}{1-s}$

588. We know that $\mathcal{L}[y(t)] = Y(s)$ and that $\mathcal{L}\left[\frac{dy}{dt}\right] = -y(0) + sY(s)$. Take the Laplace transform of the differential equation $\frac{dy}{dt} = 5y + 2e^{-3t}$ with $y(0) = 4$ and solve for $Y(s)$.

(a) $Y(s) = \frac{2}{(-s-3)(s-5)} + \frac{4}{s-5}$

(b) $Y(s) = \frac{2}{(-s-3)(s-5)} + \frac{4}{s+5}$

(c) $Y(s) = \frac{2}{(s+3)(s-5)} + \frac{4}{s-5}$

(d) $Y(s) = \frac{2}{(s+3)(s+5)} + \frac{4}{s+5}$

589. Suppose that after taking the Laplace transform of a differential equation we solve for the function $Y(s) = \frac{3}{(s+2)(s-6)}$. This is equivalent to which of the following?

(a) $Y(s) = \frac{-3/8}{s+2} + \frac{3/8}{s-6}$

(b) $Y(s) = \frac{-8/3}{s+2} + \frac{8/3}{s-6}$

(c) $Y(s) = \frac{3/8}{s+2} + \frac{-3/8}{s-6}$

(d) $Y(s) = \frac{8/3}{s+2} + \frac{-8/3}{s-6}$

590. Find $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = 3e^{-2t}e^{6t}$

(b) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = -\frac{3}{8}e^{-2t} + \frac{3}{8}e^{6t}$

(c) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = \frac{3}{8}e^{2t} - \frac{3}{8}e^{-6t}$

(d) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = e^{-3/8}e^{-2t} + e^{3/8}e^{6t}$

(e) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = -\frac{3}{8}e^{2t} + \frac{3}{8}e^{-6t}$

591. Solve the differential equation $\frac{dy}{dt} = -4y + 3e^{2t}$ with $y(0) = 5$ using the method of Laplace transforms: First take the Laplace transform of the entire equation, then solve for $Y(s)$, use the method of partial fractions to simplify the result, and take the inverse transform to get the function $y(t)$.

(a) $y(t) = \frac{9}{2}e^{4t} + \frac{1}{2}e^{-2t}$

(b) $y(t) = \frac{7}{2}e^{4t} + \frac{3}{2}e^{-2t}$

(c) $y(t) = \frac{9}{2}e^{-4t} + \frac{1}{2}e^{2t}$

(d) $y(t) = \frac{7}{2}e^{-4t} + \frac{3}{2}e^{2t}$

592. Consider the Heaviside function, which is defined as

$$u_a(t) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t \geq a \end{cases}$$

Suppose $a = 2$. What is the Laplace transform of this $\mathcal{L}[u_2(t)]$? Hint: It may be helpful to break this integral into two pieces, from 0 to 2 and from 2 to ∞.

(a) $\mathcal{L}[u_2(t)] = -\frac{1}{s}e^{2s} + \frac{1}{s}$

(b) $\mathcal{L}[u_2(t)] = \frac{1}{s}e^{-2s} - \frac{1}{s}$

(c) $\mathcal{L}[u_2(t)] = -\frac{1}{s}e^{-2s}$

(d) $\mathcal{L}[u_2(t)] = \frac{1}{s}e^{-2s}$

(e) $\mathcal{L}[u_2(t)] = \frac{1}{s}e^{2s}$

593. Find the Laplace transform of the function $u_5(t)e^{-(t-5)}$.

(a) $\mathcal{L}[u_5(t)e^{-(t-5)}] = \frac{e^5}{s+1}$

(b) $\mathcal{L}[u_5(t)e^{-(t-5)}] = \frac{e^{-5s}}{s+1}$

(c) $\mathcal{L}[u_5(t)e^{-(t-5)}] = \frac{e^5e^{-5s}}{s+1}$
(d) $\mathcal{L}[u_5(t)e^{-(t-5)}] = \frac{1}{s+1}$

594. Find the Laplace transform of the function $u_4(t)e^{3(t-4)}$.

(a) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{-3s}}{s-4}$
(b) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{3s}}{s+4}$
(c) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{-4s}}{s+3}$
(d) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{-4s}}{s-3}$
(e) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{4s}}{s-3}$

595. Find $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_2(t)e^{-2(t-3)}$
(b) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_2(t)e^{-3(t-2)}$
(c) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_2(t)e^{-3(t-3)}$
(d) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_3(t)e^{-2(t-3)}$
(e) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_3(t)e^{-2(t-2)}$
(f) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_3(t)e^{-3(t-2)}$

596. Use Laplace transforms to solve the differential equation $\frac{dy}{dt} = -2y + 4u_2(t)$ with $y(0) = 1$.

(a) $y(t) = e^{-2t} + 2u_2(t)e^{-(t-2)} - 2u_2(t)e^{-2(t-2)}$
(b) $y(t) = e^{-2t} + 4u_2(t)e^{-(t-2)} - 4u_2(t)e^{-2(t-2)}$
(c) $y(t) = e^{-2t} + 2u_2(t) - 2u_2(t)e^{-2(t-2)}$
(d) $y(t) = e^{-2t} + 4u_2(t) - 4u_2(t)e^{-2(t-2)}$

597. What is the Laplace transform of the second derivative, $\mathcal{L}\left[\frac{d^2y}{dt^2}\right]$?

(a) $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = -y'(0) + s\mathcal{L}[y]$

(b) $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = -y'(0) - sy(0) + s^2\mathcal{L}[y]$

(c) $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = -y'(0) - sy(0) + s\mathcal{L}[y]$

(d) $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = -y'(0) + sy(0) - s^2\mathcal{L}[y]$

598. We know that the differential equation $y'' = -y$ with the initial condition $y(0) = 0$ and $y'(0) = 1$ is solved by the function $y = \sin t$. By taking the Laplace transform of this equation find an expression for $\mathcal{L}[\sin t]$.

(a) $\mathcal{L}[\sin t] = \frac{1}{s^2+1}$

(b) $\mathcal{L}[\sin t] = \frac{1}{s^2-1}$

(c) $\mathcal{L}[\sin t] = \frac{s}{s^2+1}$

(d) $\mathcal{L}[\sin t] = \frac{s}{s^2-1}$

599. Find an expression for $\mathcal{L}[\sin \omega t]$, by first using your knowledge of the second order differential equation and initial conditions $y(0)$ and $y'(0)$ that are solved by this function, and then taking the Laplace transform of this equation.

(a) $\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2+1}$

(b) $\mathcal{L}[\sin \omega t] = \frac{1}{s^2\omega^2+\omega^2}$

(c) $\mathcal{L}[\sin \omega t] = \frac{1}{s^2+\omega^2}$

(d) $\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2+\omega}$

(e) $\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2+\omega^2}$

600. Find an expression for $\mathcal{L}[\cos \omega t]$, by first using your knowledge of the second order differential equation and initial conditions $y(0)$ and $y'(0)$ that are solved by this function, and then taking the Laplace transform of this equation.

(a) $\mathcal{L}[\cos \omega t] = \frac{s}{s^2+\omega^2}$

(b) $\mathcal{L}[\cos \omega t] = \frac{s\omega}{s^2+\omega^2}$

(c) $\mathcal{L}[\cos \omega t] = \frac{s\omega}{s^2-\omega^2}$

(d) $\mathcal{L}[\cos \omega t] = \frac{\omega}{s^2+\omega^2}$

(e) $\mathcal{L}[\cos \omega t] = \frac{\omega}{s^2-\omega^2}$

601. Find $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 3\cos 2t$.

(b) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 3\cos 4t$.

(c) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 4\cos 3t$.

(d) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 3\sin 2t$.

(e) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 3\sin 4t$.

(f) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 4\sin 3t$.

602. Find $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = 5\sin\sqrt{3}t$.

(b) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = 5\sin\sqrt{6}t$.

(c) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = \frac{5}{2}\sin\sqrt{3}t$.

(d) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = \frac{5}{2}\sin\sqrt{6}t$.

(e) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = \frac{5}{2\sqrt{3}}\sin\sqrt{3}t$.

603. We know that the function $f(t)$ has a Laplace transform $F(s)$, so that $\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt = F(s)$. What can we say about the Laplace transform of the product $\mathcal{L}[e^{at}f(t)]$?

(a) $\mathcal{L}[e^{at}f(t)] = e^{at}F(s)$.

(b) $\mathcal{L}[e^{at}f(t)] = e^{as}F(s)$.

(c) $\mathcal{L}[e^{at}f(t)] = F(s-a)$.

(d) $\mathcal{L}[e^{at}f(t)] = F(s+a)$.

(e) We cannot make a general statement about this Laplace transform without knowing $f(t)$.

604. Find $\mathcal{L}[3e^{4t}\sin 5t]$.

(a) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{3}{(s-4)^2+25}$.

(b) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{15}{(s-4)^2+25}$.

(c) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{15}{(s+4)^2+25}$.

(d) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{3\sqrt{5}}{(s-4)^2+5}$.

(e) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{3\sqrt{5}}{(s+4)^2+5}$.

605. Find $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right] = 2e^{-3t}\sin 4t$

(b) $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right] = 2e^{-4t}\sin 3t$

(c) $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right] = 8e^{-3t}\sin 4t$

(d) $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right] = 2e^{3t}\sin 4t$

606. $s^2+6s+15$ is equivalent to which of the following?

(a) $s^2+6s+15=(s+2)^2+11$

(b) $s^2+6s+15=(s+3)^2+6$

(c) $s^2+6s+15=(s+5)^2-10$

(d) $s^2+6s+15=(s+6)^2-21$

607. Find $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right]$, by first completing the square in the denominator.

(a) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = 2e^{-3t}\sin\sqrt{6}t$

(b) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = 2e^{3t}\cos\sqrt{6}t$

(c) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t$

(d) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = 2e^{3t}\sin\sqrt{6}t$

(e) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = e^{6t}\cos\sqrt{6}t$

(f) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = e^{-6t}\cos\sqrt{6}t$

608. Find $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right]$, by first completing the square in the denominator.

(a) $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t$

(b) $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t+2$

(c) $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t+2e^{-3t}\sin\sqrt{6}t$

(d) $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t+\frac{2}{\sqrt{6}}e^{-3t}\sin\sqrt{6}t$

609. Suppose that after taking the Laplace transform of a differential equation we obtain the function $Y(s)=\frac{2s}{(s^2+16)(s^2+3)}$. This is equivalent to which of the following?

(a) $Y(s) = \frac{2s/19}{s^2+16} + \frac{2s/19}{s^2+3}$

(b) $Y(s) = \frac{-2s/19}{s^2+16} + \frac{2s/19}{s^2+3}$

(c) $Y(s) = \frac{2s/13}{s^2+16} + \frac{-2s/13}{s^2+3}$

(d) $Y(s) = \frac{-2s/13}{s^2+16} + \frac{2s/13}{s^2+3}$

610. Find $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right] = -\frac{2}{13}\cos 4t + \frac{2}{13}\cos\sqrt{3}t.$

(b) $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right] = \frac{2}{13}\cos 4t - \frac{2}{13}\cos\sqrt{3}t.$

(c) $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right] = -\frac{2}{52}\sin 4t + \frac{2}{13\sqrt{3}}\sin\sqrt{3}t.$

(d) $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right] = \frac{2}{52}\sin 4t - \frac{2}{13\sqrt{3}}\sin\sqrt{3}t.$

611. Use Laplace transforms to solve the differential equation $y'' = -2y + 4\sin 3t$ if we know that $y(0) = y'(0) = 0$.

(a) $y = -\frac{6\sqrt{2}}{7}\sin\sqrt{2}t + \frac{4}{7}\sin 3t$

(b) $y = \frac{6\sqrt{2}}{7}\sin\sqrt{2}t - \frac{4}{7}\sin 3t$

(c) $y = \frac{12}{7}\sin\sqrt{2}t - \frac{12}{7}\sin 3t$

(d) $y = -\frac{12}{7}\cos\sqrt{2}t + \frac{12}{7}\cos 3t$

612. The Dirac delta function $\delta_a(t)$ is defined so that $\delta_a(t) = 0$ for all $t \neq a$, and if we integrate this function over any integral containing a, the result is 1. What would be $\int_5^{10}(\delta_3(t) + 2\delta_6(t) - 3\delta_8(t) + 5\delta_{11}(t))\,dt$?

(a) -1

(b) 0

(c) 4

(d) 5

(e) -30

(f) This integral cannot be determined.

613. Find $\int_5^{10}(\delta_6(t)\cdot\delta_7(t)\cdot\delta_8(t)\cdot\delta_9(t))\,dt$?

(a) -1
(b) 0
(c) 1
(d) 4
(e) 5
(f) This integral cannot be determined.

614. Find $\mathcal{L}[\delta_7(t)]$.

(a) 1
(b) 7
(c) e^{-7s}
(d) e^{-7t}
(e) This integral cannot be determined.

615. Find $\mathcal{L}[\delta_0(t)]$.

(a) 0
(b) 1
(c) e^{-s}
(d) e^{-t}
(e) This integral cannot be determined.

616. We have a system modeled as an undamped harmonic oscillator, that begins at equilibrium and at rest, so $y(0) = y'(0) = 0$, and that receives an impulse force at $t = 4$, so that it is modeled with the equation $y'' = -9y + \delta_4(t)$. Find $y(t)$.

(a) $y(t) = u_4(t)\sin 3t$
(b) $y(t) = \frac{1}{3}u_4(t)\sin 3t$
(c) $y(t) = u_4(t)\sin 3(t-4)$
(d) $y(t) = \frac{1}{3}u_4(t)\sin 3(t-4)$

617. We have a system modeled as an undamped harmonic oscillator, that begins at equilibrium and at rest, so $y(0) = y'(0) = 0$, and that receives an impulse forcing at $t = 5$, so that it is modeled with the equation $y'' = -4y + \delta_5(t)$. Find $y(t)$.

(a) $y(t) = u_5(t)\sin 2t$
(b) $y(t) = \frac{1}{2}u_5(t)\sin 2(t-5)$
(c) $y(t) = \frac{1}{2}u_2(t)\sin 2(t-5)$
(d) $y(t) = \frac{1}{5}u_2(t)\sin 5(t-2)$
(e) $y(t) = \frac{1}{5}u_5(t)\sin 2(t-2)$

618. For many differential equations, if we know how the equation responds to a nonhomogeneous forcing function that is a Dirac delta function, we can use this response function to predict how the differential equation will respond to any other forcing function and any initial conditions. For which of the following differential equations would this be impossible?

(a) $p''(q) = \frac{4}{q} + 96p' - 12p$
(b) $\frac{d^2m}{dn^2} - 100\frac{dm}{dn} + 14nm = \frac{18}{\ln(n)}$
(c) $\frac{d^2f}{dx^2} - 100\frac{df}{dx} = \frac{18}{\ln(x)}$
(d) $\frac{dc}{dr} = 12c + 3\sin(2r) + 8\cos^2(3r+2)$

619. The convolution of two functions $f * g$ is defined to be $\int_0^t f(t-u)g(u)du$. What is the convolution of the functions $f(t) = 2$ and $g(t) = e^{2t}$?

(a) $f * g = e^{2t} - 1$
(b) $f * g = 2e^{2t} - 2$
(c) $f * g = 4e^{2t} - 4$
(d) $f * g = \frac{1}{2}e^{2t} - \frac{1}{2}$
(e) This convolution cannot be computed.

620. What is the convolution of the functions $f(t) = u_2(t)$ and $g(t) = e^{-3t}$?

(a) $f * g = \frac{1}{3}e^{-3(t-2)} - \frac{1}{3}$
(b) $f * g = \frac{1}{3}e^{-3t} - \frac{1}{3}$
(c) $f * g = -\frac{1}{3}e^{-3(t-2)} + \frac{1}{3}$
(d) $f * g = -\frac{1}{3}e^{-3t} + \frac{1}{3}$
(e) This convolution cannot be computed.

621. Let $\zeta(t)$ be the solution to the initial-value problem $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \delta_0(t)$, with $y(0) = y'(0) = 0^-$. Find an expression for $\mathcal{L}[\zeta]$.

(a) $\mathcal{L}[\zeta] = 0$

(b) $\mathcal{L}[\zeta] = s^2 + ps + q$

(c) $\mathcal{L}[\zeta] = \frac{1}{s^2+ps+q}$

(d) $\mathcal{L}[\zeta] = \frac{e^{-s}}{s^2+ps+q}$

622. Let $\zeta(t) = 2e^{-3t}$ be the solution to an initial-value problem $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \delta_0(t)$, with $y(0) = y'(0) = 0^-$ for some specific values of p and q, so that $\mathcal{L}[\zeta] = \frac{1}{s^2+ps+q}$. What will be the solution of $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = 0$ if $y(0) = 0$ and $y'(0) = 5$?

(a) $y = 2e^{-3t}$

(b) $y = 10e^{-3t}$

(c) $y = \frac{2}{5}e^{-3t}$

(d) $y = 2e^{-15t}$

(e) It cannot be determined from the information given.

623. Let $\zeta(t)$ be the solution to the initial-value problem $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \delta_0(t)$, with $y(0) = y'(0) = 0^-$. Now suppose we want to solve this problem for some other forcing function $f(t)$, so that we have $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = f(t)$. Which of the following is a correct expression for $\mathcal{L}[y]$?

(a) $\mathcal{L}[y] = 0$.

(b) $\mathcal{L}[y] = \mathcal{L}[f]\mathcal{L}[\zeta]$.

(c) $\mathcal{L}[y] = \frac{\mathcal{L}[f]}{\mathcal{L}[\zeta]}$.

(d) $\mathcal{L}[y] = \mathcal{L}[f\zeta]$.

(e) None of the above

624. Let $\zeta(t) = e^{-3t}$ be the solution to an initial-value problem $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \delta_0(t)$, with $y(0) = y'(0) = 0^-$. Now suppose we want to solve this problem for the forcing function $f(t) = e^{-2t}$, so that we have $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = f(t)$. Find $y(t)$.

(a) $y(t) = e^{-2t} - e^{-3t}$

(b) $y(t) = e^{-3t} - e^{-2t}$

(c) $y(t) = \frac{e^{-2t}-e^{-3t}}{5}$

(d) $y(t) = \frac{e^{2t}-e^{-3t}}{5}$

Chapter 9: Discrete Dynamical Systems

Equilibrium and Long-Term Behavior

625. Suppose we have $a_{n+1} = 3a_n$ with $a_0 = 0$. What is a_{400}?

  (a) 0
  (b) 3^{400}
  (c) 1200
  (d) It would take way too long to figure this out.

626. Suppose we have $a_{n+1} = 3a_n$ with $a_0 = 1$. What is a_3?

  (a) 0
  (b) 1
  (c) 9
  (d) 27
  (e) None of the above

627. Suppose we have $a_{n+1} = 0.9a_n$. What is the equilibrium value?

  (a) 0
  (b) 0.9
  (c) There is no equilibrium value.
  (d) The equilibrium value cannot be determined without knowing the initial value.
  (e) None of the above

628. Suppose we have $a_{n+1} = 0.9a_n$ with $a_0 = 1$. What is a_3?

  (a) 0
  (b) 1
  (c) 0.9
  (d) 0.729
  (e) None of the above

629. Suppose we have $f_{n+1} = 3f_n - 10$ with $f_0 = 5$. What is f_3?

(a) 0
(b) 5
(c) 15
(d) 125

630. Suppose we have $f_{n+1} = 3f_n - 10$ with $f_0 = 4$. What is f_3?

(a) 0
(b) 2
(c) 4
(d) 12
(e) None of the above

631. Suppose $d_{n+1} = 0.9d_n + 2$. What is the equilibrium value?

(a) 9
(b) 2
(c) 20
(d) 20/9
(e) None of the above

632. Suppose $g_{n+1} = -2g_n + 3$. Which statement describes the long-term behavior of the solution with $g_0 = 0$.

(a) The solution stays at 0.
(b) The solution grows without bound.
(c) The solution grows and approaches the equilibrium value.
(d) The solution oscillates farther and farther from the equilibrium value.
(e) None of the above

633. **True or False** An equilibrium value can never be negative.

(a) True, and I am very confident
(b) True, but I am not very confident

(c) False, but I am not very confident
(d) False, and I am very confident

634. The following difference equation describes the population of a small town, where n is in years.

$$p_{n+1} = 0.9p_n$$

Which of the following is a true statement?

(a) Eventually this town will die out.
(b) Eventually this town will have 1,000,000 people.
(c) This town's population will fluctuate, but the town will never grow substantially or die out.
(d) A big city modeled with this difference equation would have more people in the long run than this town will have.

635. The following difference equation describes the population of a small town, where n is in years.

$$p_{n+1} = 1.17p_n$$

Which of the following is a true statement?

(a) Eventually this town will die out.
(b) Eventually this town will have 1,000,000 people.
(c) This town's population will fluctuate, but the town will never grow substantially or die out.
(d) This difference equation does not have an equilibrium value.

636. The difference equation $a_{n+1} = 1.08a_n$ might model the population of some species. If $a_0 = 5000$, which of the following statements is true?

(a) $a_{10} > 5000$
(b) $a_{40} < 5000$
(c) It is possible that $a_{30} = 5000$
(d) More than one of these statements could be true.
(e) None of these statements has to be true.

637. The difference equation $a_{n+1} = 0.93a_n$ might model the population of some species. If $a_0 = 5000$, which of the following statements is true?

(a) $a_{10} > 5000$

(b) $a_{40} < 5000$

(c) It is possible that $a_{30} = 5000$

(d) More than one of these statements could be true.

(e) None of these statements has to be true.

638. When we are looking for an equilibrium value, why can we change both a_n and a_{n+1} to E and then solve the resulting equation?

(a) a_n and a_{n+1} both represent amounts, so they're the same thing anyway.

(b) a_n and a_{n+1} are just symbols, so we can use a different symbol to represent them.

(c) At equilibrium, each term is the same as the one before.

(d) None of the above

639. A polluted lake has a percentage of its contaminants washed away each year, while factories on the lake dump in a constant amount of pollutants each year. The equilibrium value for this lake is 500 pounds. This means that

(a) if there are initially 500 pounds of contaminants, there will always be 500 pounds of contaminants (assuming the conditions remain the same).

(b) when there are 500 pounds of contaminants in the lake, the amount of pollutants being washed out each year is exactly equal to the amount being dumped in.

(c) if there are currently 750 pounds of pollutants in the lake, next year there will be fewer pollutants in the lake.

(d) All of the above are correct.

(e) Exactly two of the above are correct.

640. Suppose the only equilibrium value of a difference equation is 10, and this is a stable equilibrium. **True or False** If $a_5 = 7$, then $7 < a_8 < 10$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

641. **True or False** A difference equation can only have one equilibrium value.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

642. Consider the difference equation $a_{n+1} = ra_n$. If r is increased, what is the effect on the equilibrium value?

(a) The equilibrium value increases.
(b) The equilibrium value decreases.
(c) The equilibrium value doesn't change.
(d) The effect depends on how much r is increased by and what it started at.

643. Consider the difference equation $a_{n+1} = ra_n + b$ where $r = 3$ and $b = -2$. If r is increased, what is the effect on the equilibrium value?

(a) The equilibrium value increases.
(b) The equilibrium value decreases.
(c) The equilibrium value doesn't change.
(d) The effect depends on how much r is increased by.

Solving Homogeneous Systems of Difference Equations

644. If we are told that the general solution to a system of difference equations is

$$A_n = \begin{bmatrix} a_n \\ b_n \end{bmatrix} = c_1 \cdot (0.9)^n \begin{bmatrix} 1 \\ \frac{7}{8} \end{bmatrix} + c_2(-0.5)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix},$$

then which is an equivalent form of the solution?

(a) $a_n = c_1(0.9)^n + \frac{7}{8}c_1(0.9)^n$ and $b_n = -c_2(-0.5)^n + c_2(-0.5)^n$
(b) $a_n = c_1(0.9)^n - c_2(-0.5)^n$ and $b_n = \frac{7}{8}c_1(0.9)^n + c_2(-0.5)^n$
(c) $a_n = c_1(0.9)^n - c_1(-0.5)^n$ and $b_n = \frac{7}{8}c_2(0.9)^n + c_2(-0.5)^n$
(d) All of the above
(e) None of the above

645. The solution to a system of difference equations is

$$A_n = \begin{bmatrix} a_n \\ b_n \end{bmatrix} = c_1 \cdot (0.9)^n \begin{bmatrix} 1 \\ \frac{7}{8} \end{bmatrix} + c_2(-0.5)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

Which of the following is a true statement?

(a) This system has an unstable equilibrium.

(b) In the long-run, b will hold 7/8 of the population.

(c) The equilibrium value of this system is $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

(d) All of the above

(e) None of the above

646. If we wish to solve this system,

$$\begin{aligned} a_{n+1} &= a_n - 0.2a_n + 0.3b_n \\ b_{n+1} &= b_n - 0.3b_n \end{aligned}$$

which matrix do we need to find eigenvalues and eigenvectors for?

(a)

$$\begin{bmatrix} 1 & -0.2 & 0.3 \\ 1 & -0.3 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & -0.2 & 0.3 \\ 0 & 1 & -0.3 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 0.8 & 0.3 \\ 0.7 & 0 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0.8 & 0.3 \\ 0 & 0.7 \end{bmatrix}$$

(e) None of the above

647. In solving the system

$$\begin{aligned} a_{n+1} &= a_n - 0.2a_n + 0.3b_n \\ b_{n+1} &= b_n - 0.3b_n \end{aligned}$$

we find that the eigenvalues of the coefficient matrix are 0.8 and 0.7 with corresponding eigenvectors of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$. What is the solution to this system?

(a) $A_n = c_1(0.8)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2(0.7)^n \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

(b) $A_n = c_1(0.8)^n \begin{bmatrix} -3 \\ 1 \end{bmatrix} + c_2(0.7)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(c) $A_n = c_1(0.8) \begin{bmatrix} 1 \\ 0 \end{bmatrix}^n + c_2(0.7) \begin{bmatrix} -3 \\ 1 \end{bmatrix}^n$

(d) $A_n = c_1(0.8) \begin{bmatrix} -3 \\ 1 \end{bmatrix}^n + c_2(0.7) \begin{bmatrix} 1 \\ 0 \end{bmatrix}^n$

(e) None of the above

648. The solution to a system of difference equations is $A_n = c_1(0.8)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2(0.7)^n \begin{bmatrix} -3 \\ 1 \end{bmatrix}$. If $A_0 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, what are c_1 and c_2?

(a) $c_1 = 2$ and $c_2 = 3$

(b) $c_1 = 55/4$ and $c_2 = 110/8$

(c) $c_1 = 11$ and $c_2 = 3$

(d) $c_1 = -7$ and $c_2 = 3$

(e) None of the above

649. The following system of difference equations allows us to predict how the populations of two towns, A and B, change each year.

$$\begin{aligned} a_{n+1} &= a_n - 0.2a_n + 0.3b_n \\ b_{n+1} &= b_n - 0.3b_n \end{aligned}$$

The solution to this system is

$$A_n = c_1(0.8)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2(0.7)^n \begin{bmatrix} -3 \\ 1 \end{bmatrix}.$$

Which of the following is a true statement?

(a) This system has a stable equilibrium.

(b) In the long-run, both of these towns will be ghost towns.

(c) If there are initially 10,000 people in town B, then $b_{10} = 282$ people.

(d) All of the above

(e) None of the above

650. If $A_n = (2)^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \left(\frac{1}{3}\right)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is a solution to the system of difference equations $A_{n+1} = RA_n$, which of the following is also a solution?

(a) $(2^n) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b) $3 \cdot (2)^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 4 \cdot \left(\frac{1}{3}\right)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(c) $8 \cdot \left(\frac{1}{3}\right)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(d) All of the above

(e) None of the above

651. **True or False** If either column of the coefficient matrix of a system of homogeneous difference equations sums to a value greater than one, then the system has an unstable equilibrium.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

652. **True or False** When solving a system of two homogeneous difference equations, if one eigenvalue is greater than one and one is between 0 and 1, then one population will grow without bound while the other declines.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident