MathQuest: Series

Geometric Series

1. What will we get if we add up the infinite series of numbers: $16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$?

   (a) This infinite sum will reach a number less than 32.
   (b) This infinite sum is equal to 32.
   (c) This infinite sum will reach a number greater than 32.
   (d) Because we're adding up an infinite number of numbers which are all greater than zero, the sum diverges to infinity.

2. What will we get if we add up the infinite series of numbers: $12 + 4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \cdots$?

   (a) This infinite sum will converge to a number less than 18.
   (b) This infinite sum is equal to 18.
   (c) This infinite sum will converge a number between 18 and 19.
   (d) This infinite sum will converge a number greater than 19.
   (e) This infinite sum diverges to infinity.

3. What will we get if we add up the infinite series of numbers: $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \cdots$?

   (a) This infinite sum will converge to 1/2.
   (b) This infinite sum will converge to 2/3.
   (c) This infinite sum will converge to 2.
   (d) This is not a geometric series.

4. What will we get if we add up the first 10 terms in the series: $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \cdots$?

   (a) 0.663
   (b) 0.664
   (c) 0.666
   (d) 0.667
   (e) 0.668

5. What is $\sum_{j=1}^{5} 4j$?

   (a) 15
   (b) 20
   (c) 40
   (d) 60

6. What will we get if we add up the infinite series: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \cdots$?

   (a) 2
   (b) A number between 2 and 3.
   (c) A number between 3 and 4.
   (d) A number between 4 and 5.
   (e) A number between 5 and 10.
   (f) This infinite series diverges to infinity.

7. Which of the following series is not geometric?

   (a) $\sum_{n=0}^{\infty} \frac{15}{3^n}$
   (b) $\sum_{n=5}^{\infty} 12^{2n+4}$
   (c) $\sum_{n=1}^{\infty} 9^{-n}$
   (d) $\sum_{n=1}^{\infty} 4^{1/n}$
   (e) $\sum_{n=0}^{\infty} \frac{5 \cdot 3^n}{7^{3n}}$
   (f) More than one of these is not geometric.

8. Which of the following geometric series converge?

   (a) $\sum_{n=0}^{\infty} \frac{8}{(-2)^n}$
   (b) $\sum_{n=5}^{\infty} 6^{3n+2}$
   (c) $\sum_{n=1}^{\infty} (-4)^{-n}$
   (d) $\sum_{n=0}^{\infty} \frac{6 \cdot 2^n}{6^{3n}}$
   (e) Exactly two of these converge.
   (f) Exactly three of these converge.

9. Which of the following is/are geometric series?

(a) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

(b) $2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \cdots$

(c) $3 + 6 + 12 + 24 + \cdots$

(d) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

(e) (a) and (b) only

(f) (a),(b), and (c) only

(g) All of the above

10. $-6 + 4 - \dfrac{8}{3} + \dfrac{16}{9} - \dfrac{32}{27} =$

(a) $-\dfrac{266}{81}$

(b) $-\dfrac{422}{27}$

(c) $-\dfrac{110}{27}$

(d) $\dfrac{110}{27}$

11. $-6 + 4 - \dfrac{8}{3} + \dfrac{16}{9} - \dfrac{32}{27} + \cdots$

(a) The sum exists and equals -18

(b) The sum exists and equals $-18/5$

(c) The sum exists and equals $18/5$

(d) The sum exists and equals 18

(e) The sum does not exist

12. What happens in an infinite geometric series if the common ratio equals 1?

(a) The series has a sum, and it's equal to 1

(b) The series has a sum, but the sum depends on the first term

(c) The series does not have a sum because the partial sums don't have a limit

Convergence Tests

13. The sum of the series

$$\frac{15}{2} + \frac{45}{8} + \frac{135}{32} + \frac{405}{128} + \frac{1215}{512} + \cdots$$

 (a) Exists
 (b) Does not exist

14. The series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{10}$

 (a) Converges
 (b) Diverges

15. If $a_n \to 0$ as $n \to \infty$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

 (a) Always true
 (b) Not always true

16. If a_n is a convergent sequence, then $\displaystyle\sum_{n=1}^{\infty} a_n$ is a convergent series.

 (a) True
 (b) False

17. The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

 (a) Converges
 (b) Diverges

18. For what values of p does the series $\sum_{n=1}^{\infty} 1/n^p$ converge?

 (a) This series converges for all values of p.
 (b) This series converges only if $p > 2$.

(c) This series converges only if $p > 1$.

(d) This series converges only if $p > 0$.

(e) This series does not converge for any values of p.

19. The series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{10}{n^5}+\frac{(-3)^n}{4^n}\right)$

(a) Converges

(b) Diverges

20. The series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{1}{n}\right)$

(a) Converges

(b) Diverges

21. The series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n(1+\ln n)}$

(a) Converges

(b) Diverges

22. The series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{1}{n}\right)$

(a) Converges

(b) Diverges

23. Does the series $\sum_{n=1}^{\infty}\frac{100}{n^2+2}$ converge?

(a) Yes, this series converges.

(b) No, this series does not converge.

(c) It is impossible to tell.

24. If $a_n > b_n$ for all n and $\sum b_n$ converges, then

(a) $\sum a_n$ converges

(b) $\sum a_n$ diverges

(c) Not enough information to determine convergence or divergence of $\sum a_n$

25. The best way to test the series $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$ for convergence or divergence is

(a) Looking at the sequence of partial sums

(b) Using rules for geometric series

(c) The Integral Test

(d) Using rules for p-series

(e) The Comparison Test

(f) The Limit Comparison Test

26. Does the series $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ converge?

(a) This series converges.

(b) This series diverges.

(c) It is impossible to tell.

27. The series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$

(a) Converges

(b) Diverges

28. The series $\displaystyle\sum_{n=1}^{\infty} (n^{-1.4} + 3n^{-1.2})$

(a) Converges

(b) Diverges

29. The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{ne^n}$

(a) Converges

(b) Diverges

30. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(n-1)!}{5^n}$

(a) Converges

(b) Diverges

31. Does the series $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ converge?

(a) This series converges.

(b) This series diverges.

(c) It is impossible to tell.

32. Does the series $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$ converge?

(a) This series converges.

(b) This series diverges.

(c) It is impossible to tell.

33. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converge?

(a) This series converges.

(b) This series diverges.

(c) It is impossible to tell.

Power Series

34. Consider the power series $\sum_{n=1}^{\infty} \frac{(x-4)^n}{4^n}$. What values of x will make this series converge?

(a) This series converges for all values of x.

(b) This series converges for all values of x between 0 and 8.

(c) This series converges for all values of x between -4 and 4.

(d) This series converges for all values of x between -8 and 0.

(e) This series diverges for all values of x.

35. Consider the power series $\sum_{n=1}^{\infty} \frac{(x-4)^n}{4^n}$. Will this series converge if $x = 0$ or if $x = 8$?

(a) This series converges for both $x = 0$ and $x = 8$.
(b) This series does not converge for either $x = 0$ or $x = 8$.
(c) This series converges for $x = 8$ but does not converge for $x = 0$.
(d) This series converges for $x = 0$ but does not converge for $x = 8$.

36. Consider the power series $\sum_{n=1}^{\infty} \frac{(3x)^n}{n^8}$. What values of x will make this series converge?

(a) This series converges for all values of x.
(b) This series converges for all values of x between -3 and 3.
(c) This series converges for all values of x between 0 and 3.
(d) This series converges for all values of x between -1/3 and 1/3.
(e) This series diverges for all values of x.

37. Consider the power series $\sum_{n=1}^{\infty} \frac{(2x)^n}{n^7}$. Will this series converge if $x = -1/2$ or if $x = +1/2$?

(a) This series converges for both $x = -1/2$ and $x = +1/2$.
(b) This series does not converge for either $x = -1/2$ or $x = +1/2$.
(c) This series converges for $x = -1/2$ but does not converge for $x = +1/2$.
(d) This series converges for $x = +1/2$ but does not converge for $x = -1/2$.

38. Consider the power series $\sum_{n=1}^{\infty} \frac{(x-8)^n}{n(-6)^n}$. What values of x will make this series converge?

(a) This series converges for all values of x.
(b) This series converges for all values of x between 2 and 14.
(c) This series converges for all values of x between -8 and 8.
(d) This series converges for all values of x between 0 and 16.
(e) This series diverges for all values of x.

39. Consider the power series $\sum_{n=1}^{\infty} \frac{(x-5)^n}{n(-3)^n}$. Will this series converge if $x = 2$ or if $x = 8$?

(a) This series converges for both $x = 2$ and $x = 8$.
(b) This series does not converge for either $x = 2$ or $x = 8$.
(c) This series converges for $x = 2$ but does not converge for $x = 8$.
(d) This series converges for $x = 8$ but does not converge for $x = 2$.

40. A power series converges when $x = 2.5, 2.7$ and 2.8, but diverges when $x = 2.1, 2.2$ and 2.9. Which of the following could be the point where the power series is centered?

(a) 2.3
(b) 2.6
(c) 2.7
(d) 2.8
(e) All of the above are possible.
(f) More than one but not all of the above are possible.

Taylor Series

41. Find the Taylor series for the function $\ln(x)$ at the point $a = 1$. (No calculators allowed.)

(a) $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \cdots$
(b) $(x-1) - (x-1)^2 + 2(x-1)^3 - 6(x-1)^4 + \cdots$
(c) $\ln(x) + \frac{1}{x}(x-1) - \frac{1}{x^2}(x-1)^2 + \frac{2}{x^3}(x-1)^3 - \frac{6}{x^4}(x-1)^4 + \cdots$
(d) $\ln(x) + \frac{1}{x}(x-1) - \frac{1}{2x^2}(x-1)^2 + \frac{1}{3x^3}(x-1)^3 - \frac{1}{4x^4}(x-1)^4 + \cdots$
(e) This is not possible.

42. If $a = 0$, what function is represented by the Taylor series $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \cdots$? (No calculators allowed.)

(a) e^x
(b) $\sin x$
(c) $\cos x$
(d) This is not a Taylor series.

43. A Taylor series converges when $x = 12, 13$ and 15, but diverges when $x = 9, 16$ and 18. Which of the following could be a, the point where the Taylor series is centered?

(a) $a = 9$
(b) $a = 11$
(c) $a = 13$
(d) $a = 15$

(e) All of the above are possible.
(f) None of the above are possible.

44. Suppose we find a Taylor series for the function $f(x)$ centered at the point $a = 5$. Where would we expect a finite number of terms from this Taylor series to probably give us a better estimate?

(a) $x = 0$
(b) $x = 3$
(c) $x = 8$
(d) There is no way to tell.

45. A Taylor series for a function $f(x)$ at $a = 10$ has a radius of convergence of 3. If we use the first 10 terms of this series to estimate $f(15)$ we will probably get

(a) an infinite result.
(b) a result which is closer to the real value of $f(15)$ than if we used 5 terms.
(c) a result which is farther from the real value of $f(15)$ than if we used 25 terms.
(d) a result which is closer to the real value of $f(15)$ than if we used 15 terms.
(e) More than one of the above are true.

46. We are given a Taylor series for a function $g(x)$ at $a = -5$, with a radius of convergence of 6. Which would give the best estimate of $g(-5)$?

(a) The first term of the Taylor series.
(b) The first 5 terms of the Taylor series.
(c) The first 10 terms of the Taylor series.
(d) The first 100 terms of the Taylor series.
(e) All would give the same result.

Fourier Series

47. Find the Fourier series on the interval $[-\pi, \pi]$ for the function $y = 2x + 3$.

(a) $2\sin x - \sin 2x + \frac{2}{3}\sin 3x - \frac{1}{2}\sin 4x + \cdots$
(b) $3 + 4\sin x - 2\sin 2x + \frac{4}{3}\sin 3x - \sin 4x + \cdots$

(c) $3 + 2\sin x - \cos x + \frac{2}{3}\sin 2x - \frac{1}{2}\cos 4x + \cdots$
(d) $3 + 2\cos x - \cos 2x + \frac{2}{3}\cos 3x - \frac{1}{2}\cos 4x + \cdots$
(e) It is not possible to create this Fourier series.

48. The Fourier Series for $f = x^3$ on the interval $[-\pi, \pi]$ contains

(a) only sines.
(b) only cosines.
(c) both sines and cosines.
(d) This is impossible.

49. The Fourier Series for $f = 3e^x$ on the interval $[-\pi, \pi]$ contains

(a) only sines.
(b) only cosines.
(c) both sines and cosines.
(d) This is impossible.

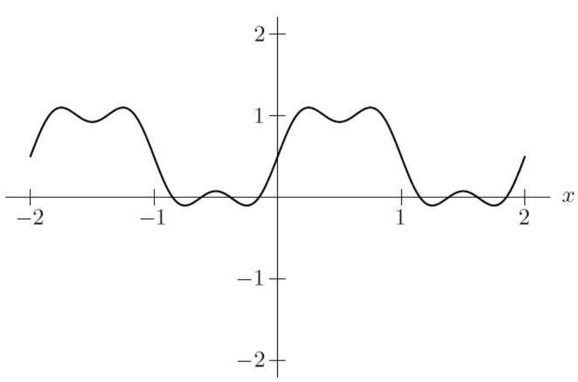
50. The figure below contains the graph of the first three terms of the Fourier series of which of the following functions?

(a) $f(x) = \begin{cases} 0, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$ and $f(x+2) = f(x)$

(b) $f(x) = \begin{cases} -1, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$ and $f(x+2) = f(x)$

(c) $f(x) = |x|$ on $-1 < x < 1$ and $f(x+2) = f(x)$

(d) $f(x) = \begin{cases} 1 + x, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$ and $f(x+2) = f(x)$



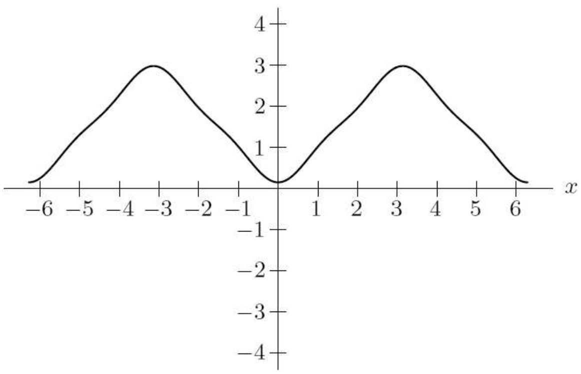
51. The figure below contains the graph of the first three terms of the Fourier series of which of the following functions?

(a) $f(x) = 3(x/\pi)^3$ on $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$

(b) $f(x) = |x|$ on $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$

(c) $f(x) = \begin{cases} -3, & -\pi < x < 0 \\ 3, & 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$

(d) $f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$



52. The figure below contains the graph of the first three non-zero terms of the Fourier series of which of the following functions?

(a) $f(x) = 3(x/\pi)^3$ on $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$

(b) $f(x) = |x|$ on $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$

(c) $f(x) = \begin{cases} -3, & -\pi < x < 0 \\ 3, & 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$

(d) $f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$